Homework 1

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1. If a three-digit decimal number is chosen at random, find the probability that exact k digits are ≥ 5, for $0 \leq k \leq 3$. The most significant digit is allowed to be 0.

   Answer: Let $p(k) = \frac{3!}{k!(3-k)!}(0.5)^k(0.5)^{3-k}$ for $0 \leq k \leq 3$.

2. A box with 15 integrated circuit chips contain 5 defectives. If a random sample of three chips is drawn, what is the probability that all three are defective?

   Answer: Probability is $\left(\frac{5}{15}\right)\left(\frac{4}{14}\right)\left(\frac{3}{13}\right)$.

3. Consider four computer firms A,B,C, D bidding for a certain contract. A survey of past bidding of these firms on similar contracts shows the following probabilities of winning:

   $$P(A) = 0.35, P(B) = 0.15, P(C) = 0.3, P(D) = 0.2.$$

   Before the decision is made to award the contract, firm B withdraws it bid. Find the new probabilities of winning the bid for A,C, and D.

   Answer: Let E be the event that either A, C and D win. Then $P(E) = 0.85.$. We have:

   $$P'(A) = 0.35/0.85, P'(C) = 0.3/0.85, P'(D) = 0.2/0.85.$$

4. A certain firm has plants A,B, and C producing 35%, 15% and 50% of the total output respectively. The probability of a non-defective product are $0.75, 0.95$ and 0.85 respectively. A customer received a defective product, what is the probability that it came from C?

   Answer: $P(defect) = 0.35 * 0.25 + 0.15 * 0.05 + 0.5 * 0.15$. We want to find $P(C|defect)$, which is:

   $$P(C|defect) = P(C\, and\, defect)/P(defect) = (0.15 * 0.05)/P(E)$$

5. In manufacturing a certain component, two types of defects are likely to occur with respective probabilities 0.05 and 0.1. What is the probability that a randomly chosen component:

(a) Does not have either kinds of defects?
Answer: $P(\text{no defect}) = 1 - (0.05 + 0.1 - 0.05 * 0.1) = 0.855$.

(b) Is defective?
Answer: $P(\text{defective}) = 0.05 + 0.1 - 0.05 * 0.1 = 0.145$.

(c) Has only one kind of defect, given that it is found to be defective?
Answer:

$$\begin{aligned}
P(\text{one kind of defect}|\text{defective}) &= \frac{P(\text{one kind of defect AND defective})}{P(\text{defective})} \\
&= \frac{P(D_1)P(\text{not } D_2) + P(D_2)P(\text{not } D_1)}{P(\text{defective})} \\
&= \frac{0.05 * 0.9 + 0.1 * 0.95}{0.145} \\
&= 0.9655.
\end{aligned}$$

6. In 3 boxes there are capacitors as shown in the following table:

Value in μF	Box 1	Box 2	Box 3
1.0	10	90	25
0.1	50	30	80
0.01	70	90	120

An experiment consists of first randomly selecting a box and then randomly selecting a capacitor from the chosen box.

(a) Prob[select a $0.1\mu F$|box 3 is chosen].
Answer: $\frac{80}{225}$.

(b) Prob[a capacitor is from box 1|selected a $0.1\mu F$].
Answer: $\frac{1/3*50/130}{1/3*50/130+1/3*30/210+1/3*80/225}$.

(c) List all nine conditional probabilities of capacitor selections, given a box selection.
Answer: use similar approach as stated above.