Tracking Triadic Cardinality Distributions for Burst Detection in Social Activity Streams

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ABSTRACT

In everyday life, we often observe unusually frequent interactions among people before or during important events, e.g., we receive/send more greetings from/to our friends on Christmas Day, than usual. We also observe that some videos suddenly go viral through people’s sharing in online social networks (OSNs). Do these seemingly different phenomena share a common structure?

All these phenomena are associated with sudden surges of user activities in networks, which we call “bursts” in this work. We find that the emergence of a burst is accompanied with the formation of triangles in networks. This finding motivates us to propose a new method to detect bursts in OSNs. We first introduce a new measure, “triadic cardinality distribution”, corresponding to the fractions of nodes with different numbers of triangles, i.e., triadic cardinalities, within a network. We demonstrate that this distribution changes when a burst occurs, and is naturally immunized against spamming social-bot attacks. Hence, by tracking triadic cardinality distributions, we can reliably detect bursts in OSNs. To avoid handling massive activity data generated by OSN users, we design an efficient sample-estimate solution to estimate the triadic cardinality distribution from sampled data. Extensive experiments conducted on real data demonstrate the usefulness of this triadic cardinality distribution and the effectiveness of our sample-estimate solution.

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1. INTRODUCTION

Online social networks (OSNs) have become ubiquitous platforms that provide various ways for users to interact over the Internet, such as tweeting tweets, sharing links, messaging friends, commenting on posts, and mentioning/replying to other users (i.e., @someone). When intense user interactions take place in a short time period, there will be a surge in the volume of user activities in an OSN. Such a surge of user activity, which we call a burst in this work, usually relates to emergent events that are occurring or about to occur in the real world. For example, Michael Jackson’s death on June 25, 2009 triggered a global outpouring of grief on Twitter [15], and the event even crashed Twitter for several minutes [30]. In addition to bursts caused by real world events, some bursts arising from OSNs can also cause enormous social impact in the real world. For example, the 2011 England riots, in which people used OSNs to organize, resulted in 3,443 crimes across London due to this disorder [1]. Hence, detecting bursts in OSNs is an important task, both for OSN managers to monitor the operation status of an OSN, and for government agencies to anticipate any emergent social disorder.

Typically, there are two types of user interactions in OSNs. First is the interaction between users (we refer to this as user-user interaction), e.g., a user sends a message to another user, while the second is the interaction between a user and a media content piece (we refer to this as user-content interaction), e.g., a user posts a video link. Examples of bursts caused by these two types of interactions include, many greetings being sent/received among people on Christmas Day, and videos suddenly becoming viral after one day of sharing in an OSN. At first sight, detecting such bursts in an OSN is not difficult. For example, a straightforward way to detect bursts caused by user-user interactions is to count the number of pairwise user interactions within a time window, and report a burst if the volume lies above a given threshold. However, this method is vulnerable to spamming social-bot attacks [10, 14, 31, 7, 33, 6], which can suddenly generate a huge amount of spamming interactions in the OSN. Hence, this method can result in many false alarms due to the existence of social bots. Similar problem also exist when detecting bursts caused by user-content interactions. Many previous works on burst detection are based on idealistic assumptions [17, 39, 24, 13] and simply ignore the existence of social bots.

Present work. The primary goal of this work is to leverage a special triangle structure, which is a feature of humans, to design a robust burst detection method that is immunized against common social-bot attacks. We first describe the
triangle structure shared by both types of user interactions.

Interaction triangles in user-user interactions. Humans form social networks with larger clustering coefficients than those in random networks [38] because social networks exhibit many triadic closures [19]. This is due to the social phenomenon of “friends of my friends are also my friends”. Since user-user interactions usually take place along social links, this property implies that user-user interactions should also exhibit many triadic closures (which we will verify in experiments). In other words, when a group of users suddenly becomes active, or we say an interaction burst occurs, in addition to observing the rise of volume of pairwise interactions, we expect to also observe many interactions among three neighboring users, i.e., many interaction triangles form if we consider an edge of an interaction triangle to be a user-user interaction. This is illustrated in Fig. 1(a) when no interaction burst occurs, while in Fig. 1(b), an interaction burst occurs. In contrast, activities generated by social bots do not possess many triangles since social bots typically select their targets randomly from an OSN [7, 33].

Influence triangles in user-content interactions. We say that a media content piece becomes bursty if many users interact with it in a short time period. There are many reasons why a user interacts with a piece of media content. Here, we are particularly interested in the case where one user influences another user to interact with the content, a.k.a. the cascading diffusion [21] or word-of-mouth spreading [27]. It is known that many emerging news stories arising from OSNs are related to this mechanism such as the story about the killing of Osama bin Laden [34]. We find that a bursty media content piece formed by this mechanism is associated with triangle formations in a network. To illustrate this, consider Fig. 2(a), in which there are five user nodes \{a, b, d, e, u\} and four content nodes \{c_1, c_2, c_3, c_4\}. A directed edge between two users means that one follows the other, and an undirected edge labeled with a timestamp between a user node and a content node represents an interaction between the user and the content at the labeled time. We say content node \(c\) has an influence triangle if there exist two users \(a, b\) such that \(a\) follows \(b\) and \(b\) interacts with \(c\) later than \(b\) does. In other words, the reason \(a\) interacts with \(c\) is due to the influence of \(b\) on \(a\). In Fig. 2(a), only \(c_2\) has an influence triangle, the others have no influence triangle, meaning that the majority of user-content interactions are not due to influence; while in Fig. 2(b), every content node is part of at least one influence triangle, meaning that many content pieces are spreading in a cascading manner in the OSN. From the perspective of an OSN manager who wants to know the operation status of the OSN, if the OSN suddenly switches to a state similar to Fig. 2(a) (from a previous state similar to Fig. 2(a)), he knows that a cascading burst is present in the OSN.
2. PROBLEM FORMULATION

We first formally define the notion of social activity stream as mentioned in previous section. Then we define triadic cardinality distribution and describe our proposed solution.

2.1 Social Activity Stream

We represent an OSN by \( G(V, E, C) \), where \( V \) is a set of users, \( E \) is a set of relationships among users, and \( C \) is a set of media content such as hashtags and video links. Here, a relationship between two users can be undirected like the friend relationship in Facebook, or directed like the follower relationship in Twitter.

Users in the OSN generate social activities, e.g., interact with other users in \( V \), or content in \( C \). We denote a social activity by \( a \in V \times (V \cup C) \times [0, \infty) \). Here user-user interaction, \( a = (u, v, t) \), corresponds to user \( u \) interacting with user \( v \) at time \( t \); and user-content interaction, \( a = (u, c, t) \), corresponds to user \( u \) interacting with content \( c \) at time \( t \). These social activities are aggregated chronologically to form a social activity stream, denoted by \( S = \{a_1, a_2, \ldots\} \), where \( a_k \) denotes the \( k \)-th social activity in the stream.

2.2 Triadic Cardinality Distribution

Triadic cardinality distributions are defined on two interaction multi-graphs which are formed by user-user and user-content interactions, respectively.

Interaction multi-graphs. Within a time window (e.g., an hour, a day or a week), user-user interactions in stream \( S \) form a multi-graph \( G_{uu}(V, E_{uu}) \), where \( V \) is the original set of users, and \( E_{uu} \) is a multi-set consisting of user-user interactions in the window. The triadic cardinality of a user \( u \in V \) is the number of interaction triangles related to \( u \) in \( G_{uu} \). For example, user \( u \) in Fig. 1(a) has triadic cardinality two, and all other users have triadic cardinality one.

Similarly, user-content interactions also form a multi-graph \( G_{uc}(V \cup C, E_{uc}) \) in a time window. Unlike \( G_{uu} \), the node set includes both user nodes \( V \) and content nodes \( C \), and the edge set includes user relations \( E \) and a multi-set \( E_{uc} \) denoting user-content interactions in the window. Note that in \( G_{uc} \), triadic cardinality is only defined for content nodes, and the triadic cardinality of a content node \( c \in C \) is the number of influence triangles related to \( c \) in \( G_{uc} \). For example, in Fig. 2(a), content \( c_2 \) has triadic cardinality one, and all other content nodes have triadic cardinality zero.

Triadic cardinality distribution. Let \( \theta = (\theta_0, \ldots, \theta_{|V|}) \) and \( \vartheta = (\vartheta_0, \ldots, \vartheta_{|V|}) \) denote the triadic cardinality distributions on \( G_{uu} \) and \( G_{uc} \) respectively. Here, \( \theta_i \) (\( \vartheta_i \)) is the fraction of users (content pieces) with triadic cardinality \( i \), and \( W(W') \) is the maximum triadic cardinality in \( G_{uu} (G_{uc}) \).

The importance of the triadic cardinality distribution lies in its capability of providing succinct summary information to characterize burst patterns in a large scale OSN. By tracking triadic cardinality distributions, we will discover burst occurrences in an OSN.

2.3 Overview of Our Solution

We propose an on-line solution capable of tracking the triadic cardinality distribution from a high-speed social activity stream, as illustrated in Fig. 3.

Our solution consists of sampling a social activity stream in a time window maintaining only summary statistics, and constructing an estimate of the triadic cardinality distribution from the summary statistics at the end of a time window. The advantage of this approach is that it is unnecessary to store all of the samples in the stream, and enables us to detect bursts in a near-real-time fashion.

3. STREAM SAMPLING METHOD

In this section, we introduce the sampling method in our solution. The purpose of sampling is to reduce the computational cost in handling the massive amount of data in a high-speed social activity stream.

3.1 Sampling Stream with a Coin

The stream sampling method works as follows. We toss a biased coin for each social activity \( a \in S \). We keep \( a \) with probability \( p \), and ignore it with probability \( 1 - p \). Hence, each social activity is independently sampled, and at the end of the time window, only a fraction \( p \) of the stream is kept. We use these samples to obtain a summary statistics of the stream in the current window, which we describe later.

3.2 Probability of Sampling a Triangle

When social activities in the stream are sampled, triangles in \( G_{uu} \) and \( G_{uc} \) are sampled accordingly. Obviously, the probability of sampling a triangle depends on \( p \). In what follows, we analyze the relationship between triangle sampling probability and \( p \), for an interaction triangle and an influence triangle, respectively.

Probability of sampling an interaction triangle. Sampling an interaction triangle, which consists of three user-user interaction edges in \( E_{uu} \), is equivalent to all its three edges being sampled. Because each interaction edge is independently sampled with probability \( p \), then an interaction triangle is sampled with probability \( p^3 \), as illustrated in Fig. 4(a).

Probability of sampling an influence triangle. Calculating the probability of sampling an influence triangle is more complicated. First, we know that an influence triangle consists of two user-content interaction edges in \( E_{uc} \) and one social edge in \( E \). Second, we note that stream sampling only applies to edges in \( E_{uc} \). Therefore, edges in \( E_{uu} \) are not sampled as they do not appear in the social activity stream.

In Fig. 4(b), suppose we have sampled two user-content interaction edges \( uc \) and \( vc \), and assume user \( u \) interacted
with content c earlier than user u. To determine whether content c has an influence triangle formed by u and v, we need to check whether (directed) edge (u, v) exists in \( E \).

This can be done by querying neighbors of one of the two users in the OSN. For example, in Twitter, we query followers of v and check whether v follows u; or in Facebook, we query friends of v and check whether u is a friend of v.

Suppose we observe \( n_c \) sampled users that all interact with c during the current time window, denoted by \( V_c = \{ u_1, \ldots, u_{n_c} \} \) where \( u_i \) interacted with c earlier than \( u_j \), \( i < j \). To verify every sampled triangle related to c, we need to query the OSN \( n_c(n_c - 1)/2 \) times. This query cost is obviously expensive when \( n_c \) is large. To reduce this query cost, instead of checking every possible user pair, we check a user pair with probability \( p' \). This is equivalent to sampling a social edge in \( E \) with probability \( p' \), conditioned on the two associated user-content interactions having been sampled. Then, it is easy to see that an influence triangle is sampled with probability \( p'p'^2 \).

We summarize the above discussions in Theorem 1.

**Theorem 1.** If we independently sample each social activity in stream S with probability \( p \), and check the existence of a user relation in the OSN with probability \( p' \), then each interaction (influence) triangle in graph \( G_{uu} (G_{uc}) \) is sampled with identical probability

\[
p_s = \begin{cases} 
p^3 & \text{for an interaction triangle}, 
p'p'^2 & \text{for an influence triangle}. \end{cases}
\]

**Remark.** Although triangles of the same type are sampled identically, they may not be sampled independently, such as the cases two triangles have shared edges in Fig. 4(c). We will consider this issue in detail in Section 4.

### 3.3 Statistics of Sampled Data

The above sampling process is equivalent to sampling edges in multi-graphs \( G_{uu} \) and \( G_{uc} \): an activity edge \( e \in E_{uu} \cup E_{uc} \) is independently sampled with probability \( p \); a social edge \( e' \in E \) is sampled with conditional probability \( p' \).

At the end of the time window, we obtain two sampled multi-graphs \( G_{uu} \) and \( G_{uc} \). Calculating the triadic cardinalities for nodes in these reduced graphs is much easier than on the original unsampled graphs. For \( G_{uu} \), we calculate triadic cardinality for each user node, and obtain statistics \( g = (g_0, \ldots, g_M) \), where \( g_0, 0 \leq j \leq M \), denotes the number of nodes with \( j \) triangles in \( G_{uu} \). Similar statistics are also obtained from \( G_{uc} \), denoted by \( f = (f_0, \ldots, f_M) \) (where \( f_j \) is the number of content nodes with \( j \) influence triangles in \( G_{uc} \)). We only need to store \( g \) and \( f \) in computer memory and use them to estimate \( \theta \) and \( \vartheta \) in the next section.

### 4. ESTIMATION METHODS

We are now ready to derive a maximum likelihood estimate (MLE) of the triadic cardinality distribution using statistics obtained in the sampling step. The estimation in this section can be viewed as an analog of network flow size distribution estimation [11, 26] in which a packet in a flow is viewed to be a triangle of a node. However, in our case,

\[ \text{triangle samples are not independent, and a node may have no triangles. These issues complicate estimation, and we will describe how to solve these issues in this section.} \]

Note that we only discuss how to obtain the MLE of \( \theta \) using \( u \), as the MLE of \( \vartheta \) using \( f \) is easily obtained using a similar approach. To estimate \( \theta \), we first consider the easier case where graph size \( |V| = n \) is known. Later, we extend our analysis to the case where \( |V| \) is unknown.

#### 4.1 MLE when Graph Size is Known

Recall that \( g_j, 0 \leq j \leq M \), is the number of nodes with \( j \) sampled triangles in \( G_{uu} \). First, note that observing a node with \( j \) sampled triangles in \( G_{uu} \) implies that the node has at least \( j \) triangles in \( G_{uu} \).

We also need to pay special attention to \( g_0 \), which is the number of nodes with no triangle in \( G_{uu} \). Due to sampling, some nodes may be unobserved (e.g., no edge attached to the node is sampled), and these unobserved nodes also have no sampled triangle. We include these in \( g_0 \); the advantage of this inclusion will be seen later. Since we have assumed a total of \( n \) nodes in \( G_{uu} \), the number of unobserved nodes is \( n - \sum_{j=0}^{M} g_j \). Therefore, we calibrate \( g_0 \) by

\[
g_0 \triangleq n - \sum_{j=1}^{M} g_j.
\]

Our goal is to derive an MLE of \( \theta \). To this end, we need to model the sampling process. For a randomly chosen node, let \( X \) denote the number of triangles to which it belongs in \( G_{uu} \), and let \( Y \) denote the number of triangles observed during sampling. Then \( P(Y = j | X = i), 0 \leq j \leq i, \) is the conditional probability that a node has \( j \) sampled triangles in \( G_{uu} \) given that it has \( i \) triangles in \( G_{uu} \). The sampling of a triangle can be viewed as a Bernoulli trial with a success probability of \( p_s \), according to Theorem 1. If Bernoulli trials are independent, which means triangles are independently sampled, then \( P(Y = j | X = i) \) follows a binomial distribution. However, independence does not hold for triangles having shared edges, as illustrated in Fig. 4(c). As a result, it is non-trivial to derive \( P(Y = j | X = i) \) with the existence of dependence. To deal with this dependence, we approximate sums of dependent Bernoulli random variables by a Beta-binomial distribution [40], which yields

\[
P(Y = j | X = i) = \text{BetaBin}(j|i,s_0/\alpha,(1-s_0)/\alpha)
\]

\[
= \left( \begin{array}{c} i \\ j \end{array} \right) \prod_{j=0}^{i-j}(s_0 + p_j) \prod_{j=0}^{i-1}(s_0 + 1 - p_j) \prod_{j=0}^{i-1}(s_0 + 1) \triangleq b_{j|}\alpha
\]

where \( \prod_{0}^{i-1} \triangleq 1 \). The above Beta-binomial distribution parameterized by \( \alpha \) allows pairwise identically distributed Bernoulli trials to have covariance \( \alpha p_0(1 - p_0)/(1 + \alpha) \). It reduces to a binomial distribution when \( \alpha = 0 \). We have carried out \( \chi^2 \) goodness-of-fit tests and the results demonstrate that the above model indeed fits well the observed data on many graphs (and is always better than the binomial model, of course).

Using this model, we easily obtain the likelihood of observing a node to have \( j \) sampled triangles, i.e.,

\[
P(Y = j) = \sum_{i=j}^{W} P(Y = j | X = i) P(X = i) = \sum_{i=j}^{W} b_{j|0} \theta_i.
\]

Then, the log-likelihood of all observations \( \{Y_k = y_k\}_{k=1}^{n} \).
where $Y_k = y_k$ denotes the $k$-th node having $y_k$ sampled triangles, yields

$$L(\theta, \alpha) \triangleq \log P(Y_k = y_k | \theta, \alpha) = \sum_{j=0}^{M} g_j \log \sum_{i=0}^{W} b_{ji}(\alpha) \theta_i. \quad (2)$$

The MLE of $\theta$ can then be obtained by maximizing (2) with respect to $\theta$ and $\alpha$ under the constraint that $\sum_{i=0}^{\infty} \theta_i = 1$. Note that this is non-trivial due to the summation inside the log operation. In the next subsection, we use the expectation-maximization (EM) algorithm to obtain the MLE in a more convenient way.

4.2 EM Algorithm when Graph Size is Known

If we already know that the $k$-th node has $x_k$ triangles in $G_{au}$, i.e., $X_k = x_k$, then the complete likelihood of observations $\{(Y_k, X_k)\}_{k=1}^{n}$ is

$$P((Y_k, X_k)_{k=1}^{n}) = \prod_{k=1}^{n} P(Y_k = y_k, X_k = x_k) = \prod_{j=0}^{M} \prod_{i=0}^{W} P(Y = j, X = i | \theta_i, \alpha)$$

where $x_k = \sum_{i=0}^{M} 1(i = k \land y_k = j)$ is the number of nodes with $i$ triangles and $j$ of them being sampled (and 1(·) is the indicator function). The complete log-likelihood is

$$L_c(\theta, \alpha) \triangleq \sum_{j=0}^{M} \sum_{i=0}^{W} x_{ij} \log [b_{ji}(\alpha) \theta_i]. \quad (3)$$

Here, we can treat $\{X_k\}_{k=1}^{n}$ as hidden variables, and apply the EM algorithm to calculate the MLE.

**E-step:** We calculate the expectation of the complete log-likelihood in Eq. (3) with respect to hidden variables $\{X_k\}_{k}$, conditioned on data $\{Y_k\}_k$ and previous estimates $\hat{\theta}^{(t)}$ and $\hat{\alpha}^{(t)}$. That is

$$Q(\theta, \alpha; \hat{\theta}^{(t)}, \hat{\alpha}^{(t)}) \triangleq \sum_{j=0}^{M} \sum_{i=0}^{W} \mathbb{E}_{\hat{\theta}^{(t)}, \hat{\alpha}^{(t)}} [x_{ij}] \log [b_{ji}(\alpha) \theta_i].$$

Here, $\mathbb{E}_{\hat{\theta}^{(t)}, \hat{\alpha}^{(t)}} [x_{ij}]$ can be viewed as the average number of nodes that have $i$ triangles in $G_{au}$, of which $j$ are sampled. Because

$$P(X = i | Y = j, \theta^{(i)}, \alpha^{(i)}) = \frac{P(Y = j | X = i, \theta^{(i)}, \alpha^{(i)}) P(X = i | \theta^{(i)})}{\sum_{i'} P(Y = j | X = i', \alpha^{(i')}) P(X = i' | \theta^{(i')})}$$

and we have observed $g_j$ nodes with $j$ sampled triangles, then $P(X = i | Y = j, \theta^{(i)}, \alpha^{(i)}) = \frac{g_j b_{ji}(\alpha^{(i)}) \theta_i^{(i)}}{\hat{p}_{ij}}$, where $\hat{p}_{ij}$ is the fraction of nodes with $i$ triangles over the nodes having at least one triangle in $G_{au}$.

**M-step:** We now maximize $Q(\theta, \alpha; \hat{\theta}^{(t)}, \hat{\alpha}^{(t)})$ with respect to $\theta$ and $\alpha$. As a result, we cannot easily model the absence of nodes by $\theta_i$, and this complicates estimation design. To solve this issue, we need to slightly reformulate our problem: (i) instead of estimating the total number of nodes in $G_{au}$, we estimate the number of nodes belonging to at the once triangle in $G_{au}$, denoted by $n_i$, and (ii) we estimate the triadic cardinality distribution $\theta^+ = (\theta^+_1, \ldots, \theta^+_n)$, where $\theta^+_i$ is the fraction of nodes with $i$ triangles over the nodes having at least one triangle in $G_{au}$.

**Estimating $n_i$**. Under the Beta-binomial model, the probability that a node has $i$ triangles in $G_{au}$, of which none are sampled, is

$$q_i(\alpha) \triangleq P(Y = 0 | X = i) = \prod_{k=0}^{i-1} \left(1 - \frac{p_k}{\alpha + 1}\right).$$

Then, the probability that a node has triangles in $G_{au}$, of which none are sampled, is

$$q(\theta^+, \alpha) \triangleq P(Y = 0 | X \geq 1) = \sum_{i=1}^{n} q_i(\alpha) \theta^+_i.$$ 

Because there are $\sum_{j=1}^{M} g_j$ nodes having been observed to have at least one sampled triangle. Hence, $n_i$ can be estimated by

$$\hat{n}_i = \frac{1}{1 - q(\theta^+, \alpha)} \sum_{j=1}^{M} g_j.$$ 

Note that estimator (4) relies on $\theta^+$ and $\alpha$, and we can obtain them using the following procedure.

**Estimating $\theta^+$ and $\alpha$.** We discard $g_0$ and only use $g^+ = (g_1, \ldots, g_M)$ to estimate $\theta^+$ and $\alpha$. The basic idea is to derive the likelihood for nodes that are observed to have at least one sampled triangle, i.e., $\{Y_k = y_k : y_k \geq 1\}$. In this case, the probability that a node has $X = i$ triangles, and $Y = j$ of them are sampled, conditioned on $Y \geq 1$, is

$$P(Y = j | X = i, Y \geq 1) = \frac{\text{BetaBin}(j | i, \frac{p_s}{\alpha}, (1 - p_s)/\alpha)}{1 - \text{BetaBin}(0 | i, \frac{p_s}{\alpha}, (1 - p_s)/\alpha)} \triangleq a_{ji}(\alpha), j \geq 1.$$
Then the probability that a node is observed to have $j$ sampled triangles, conditioned on $Y \geq 1,$ is

$$P(Y = j|Y \geq 1) = \sum_{i=j}^{W} P(Y = j|X = i, Y \geq 1)P(X = i|Y \geq 1) = \sum_{i=j}^{W} a_{ij}(\alpha)\phi_{i},$$

where

$$\phi_{i} = P(X = i|Y \geq 1) = \frac{\theta_{i}^{+}[1 - q_{i}(\alpha)]}{\sum_{i=1}^{W}\theta_{i}^{+}[1 - q_{i}(\alpha)]}, \quad i \geq 1. \quad (5)$$

Now it is straightforward to obtain the previously mentioned likelihood. Furthermore, we can leverage our previously developed EM algorithm by replacing $\theta_{i}$ by $\phi_{i}$, $b_{ij}$ by $a_{ij}$, to obtain MLEs for $\phi$ and $\alpha$. We omit these details, and directly provide the final EM iterations:

$$\hat{\phi}_{i}^{(t+1)} = \frac{\sum_{j=1}^{W} E_{\phi^{(t)}, \alpha^{(t)}}[z_{ij}]}{\sum_{j=1}^{W} \sum_{i=j}^{W} E_{\phi^{(t)}, \alpha^{(t)}}[z_{ij}]} \quad \hat{\alpha}_{i}^{(t+1)} = \arg \max_{\alpha} Q(\phi, \alpha; \phi^{(t)}, \alpha^{(t)}) \quad a \text{ solved using gradient decent methods.}$$

Once EM converges, we obtain estimates $\hat{\phi}$ and $\hat{\alpha}$. The estimate for $\theta^{+}$ is then obtained by Eq. (5), i.e.,

$$\hat{\theta}_{i}^{+} = \frac{\hat{\phi}_{i}([1 - q_{i}(\hat{\alpha})]}{\sum_{i=1}^{W}\hat{\phi}_{i}([1 - q_{i}(\hat{\alpha})]}}, \quad 1 \leq i \leq W. \quad (6)$$

Finally, $\hat{n}_{+}$ is obtained by the estimator in Eq. (4).

5. EXPERIMENTS

In this section, we first empirically verify the claims we have made. Then, we validate the proposed estimation methods on several real-world networks. Finally, we illustrate our method to detect bursts in Twitter during the 2014 Hong Kong Occupy Central movement.

5.1 Analyzing Bursts in Enron Dataset

In the first experiment, we use a public email communication dataset to empirically show how bursts in networks can change the triadic cardinality distribution, and verify our claims previously made.

Enron email dataset. The Enron email dataset [18] includes the entire email communications (e.g., who sent an email to whom at what time) of the Enron corporation from its startup to bankruptcy. The used dataset is carefully cleaned by removing spam accounts/emails and emails with incorrect timestamps. The cleaned dataset contains 22,477 email accounts and 164,081 email communications between Jan 2001 and Apr 2002. We use this dataset to study patterns of bursts caused by email communications among people, i.e., user-user interactions.

Observations from data. Because the data has been cleaned, the number of user-user interactions (i.e., number of sent emails$^{2}$) per time window reliably indicates burst occurrences. We show the number of emails sent per week in

\[2\text{If an email has } x \text{ recipients, we count it } x \text{ times.}\]
targets; and sends spams to each of these two targets. The spammer repeats this step a number of times.

In order to measure the extent that spams can distort the original triadic cardinality distribution of week 23, we use Kullback-Leibler (KL) divergence to measure the difference between the original and distorted distributions. The relationship between KL divergence and the number of injected spams is shown in Fig. 7(a). For both strategies, KL divergences both increase as more spams are injected into the interaction network, which is expected. The Random-Friend strategy can cause larger divergences than the Random strategy, as Random-Friend strategy is easier to introduce new triangles to the interaction network of week 23 for the reason that two friends are more likely to communicate in a week. However, even when 10^4 spams are injected, the spams incur an increasing KL divergence of less than 0.04. From Fig. 7(b), we can see that the divergence is indeed small. (This may be explained by the “center of attention” phenomenon [3], i.e., a person may have hundreds of friends but he usually only interacts with a small fraction of them in a time window. Hence, Random-Friend strategy does not form many triangles.) Therefore, these observations verify that triadic cardinality distribution is robust against common spamming attacks.

To “correct” this distribution to approach the ground truth distribution, we sample an edge with probability p and obtain a sampled graph. Then we calculate the triadic cardinality for each node in the sampled graph, and obtain statistics g. Note that the estimator uses g to obtain an estimate of the triadic cardinality distribution for each graph, which is then compared with the ground truth distribution, i.e., the triadic cardinality distribution of the original unsampled graph, to evaluate the performance of the estimation method.

Validation when graph size is known. First evaluate the estimation method when the graph size is known in advance, as is the assumption of our first method. The first method outputs estimate \( \hat{\theta} = (\theta_0, \ldots, \theta_W) \).

The estimates on the four graphs and comparisons with ground truth distributions are depicted in Fig. 8. For each graph, we set p = 0.05, 0.1 and 0.15, respectively. From these results, we show that when more data is sampled the estimate generally improves, but even when p = 0.05 is sufficient to obtain a good estimate. The sampled triadic cardinality distribution of g for p = 0.15 is also shown for each graph. It is clear to see that the estimator has the ability to “correct” this distribution to approach the ground truth distribution.

We also compare the computational efficiency of our sampling approach against a naive method that uses all of the original graph to calculate \( \hat{\theta} \) in an exact fashion. The results are depicted in Fig. 9. Obviously, the naive method is very inefficient and our sample-estimate solution is at least about 50 times faster with \( p = 0.3 \) on all of the four graphs.

### Datasets

Because the input of our estimation methods is in fact a sampled graph, we use several public available graphs of different types and scales from the SNAP graph repository (snap.stanford.edu/data) as our testbeds. We summarize statistics of these graphs in Table 1.

### Table 1: Network statistics

<table>
<thead>
<tr>
<th>Network</th>
<th>Type</th>
<th>Nodes</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>HepTh</td>
<td>directed, citation</td>
<td>27,770</td>
<td>352,807</td>
</tr>
<tr>
<td>DBLP</td>
<td>undirected, coauthor</td>
<td>317,080</td>
<td>1,049,866</td>
</tr>
<tr>
<td>YouTube</td>
<td>undirected, OSN</td>
<td>1,134,890</td>
<td>2,987,624</td>
</tr>
<tr>
<td>Pokec</td>
<td>directed, OSN</td>
<td>1,632,803</td>
<td>30,622,564</td>
</tr>
</tbody>
</table>

For each graph, we sample an edge with probability p, and obtain a sampled graph. We then calculate the triadic cardinality for each node in the sampled graph, and obtain statistics g. Note that the estimator uses g to obtain an estimate of the triadic cardinality distribution for each graph, which is then compared with the ground truth distribution, i.e., the triadic cardinality distribution of the original unsampled graph, to evaluate the performance of the estimation method.

### Figure 7: Impacts of spam.

(a) KL divergence  
(b) Distorted distributions

### Figure 8: Estimates of \( \theta \) when graph size is known.

\( \hat{\theta} \) corresponding to each graph and p is typically small, ranging from 0.0015 to 0.028. W = 10^4 and each result is averaged over 100 runs.

### Figure 9: Computational efficiency comparison
For Twitter itself, this stream is easily obtained by directly putting our solution is a social activity stream from Twitter. Building a Twitter social activity stream.

The computational efficiency is similar to results depicted in Fig. 9.

The estimate of $n_+$ for each graph is shown in Fig. 11. Because the majority of the nodes have small triadic cardinalities, good estimates of $\theta_+^i$ for small values of $i$ are critical for a good estimate of $n_+$ using estimator (4). For the HepTh graph, estimate $\hat{n}_+$ is very accurate even with small $p$. While for the other three graphs, accurate estimates of $n_+$ require relatively large sample rates, and $\hat{n}_+$ is usually an underestimate of $n_+$ on DBLP and Pokec due to a slight underestimate of $\theta^i$ for small values of $i$ on the two graphs. Nevertheless, using a sample rate $p = 0.3$, the relative estimation error for $\hat{n}_+$ is less than 20% for all four graphs. The design of a better estimator for $n_+$ is left for future work.

5.3 Application: Burst Detection in 2014 Hong Kong Occupy Central Movement

In the third experiment, we apply our solution to detect bursts in Twitter during the 2014 Hong Kong Occupy Central movement.

2014 Hong Kong Occupy Central movement a.k.a. the Umbrella Revolution, began in Sept 2014 when activists in Hong Kong protested against the government and occupied several major streets of Hong Kong to go against a decision made by China’s Standing Committee of the National People’s Congress on the proposed electoral reform. Protesters began gathering from Sept 28 on and the movement was still ongoing while we were collecting the data.

Building a Twitter social activity stream. The input of our solution is a social activity stream from Twitter. For Twitter itself, this stream is easily obtained by directly aggregating tweets of users. While for third parties who do not own user’s tweets, the stream can be obtained by following users using a set of Twitter accounts, called detectors, and aggregating tweets received by detectors (i.e., detectors’ timelines) to form a social activity stream. Since the movement had already begun prior to our starting this work, we rebuilt the social activity stream by searching tweets containing at least one of the following hashtags: #OccupyCentral, #OccupyHK, #UmbrellaRevolution, #UmbrellaMovement and #UMHK, between Sept 1 and Nov 30 using Twitter search APIs. This produced 66,589 Twitter users, and these users form the detectors from whom we want to detect bursts. Next, we collect each user’s tweets between Sept 1 and Nov 30, and extract user mentions (i.e., user-user interactions) and user hashtags (i.e., user-content interactions) from tweets to form a social activity stream, with a time span of 91 days.

Settings. We set the length of a time window to be one day. In a time window, we sample each social activity with probability $p = 0.3$ and check a social relation with probability $p' = 0.3$. For interaction bursts caused by user-user interactions, because we know the user population, i.e., $n = 66,589$, we apply the first estimation method to obtain $\hat{\theta} = (\theta_1, \ldots, \theta_W)$ for each window. For cascading bursts caused by user-content interactions, as we do not know the number of hashtags in advance, we apply the second method to obtain estimates $\hat{n}_+$, i.e., the number of hashtags with at least one influence triangle, and $\hat{\theta}^+ = (\theta_1^+, \ldots, \theta_W^+)$ for each window. Combining $\hat{n}_+$ with $\hat{\theta}^+$, we use $\hat{n}_+ \hat{\theta}^+$, i.e., frequencies, to characterize patterns of user-content interactions in each window. For both $\hat{\theta}$ and $\hat{\theta}^+$, $W$ is set to be $10^4$.

Results. We first answer the question: are there significant differences for the two distributions before and during the movement? In Fig. 12, we compare the distributions before (Sept 1 to Sept 3) and during (Sept 28 to Sept 30) the movement. We can find that when the movement began on Sept 28, the distributions of the two kinds of interactions shift to the right, indicating that many interaction and influence triangles form when the movement starts. Therefore, these observations confirm our motivation for detecting bursts by tracking triadic cardinality distributions.
were studying how to characterize structures of different types of networks. Turkett et al. [36] used motifs to analyze computer network usage, and [37] proposed sampling methods to efficiently estimate motif statistics in a large graph. However, both the motivation in [36] and subgraph statistics defined in [37] are different from ours.

Recently, there are many works on estimating the number of triangles [35, 8, 25, 16, 2] or clustering coefficient [29] in a large graph. However, these methods cannot be used to estimate the triadic cardinality distribution. Becchetti et al. [5] used a min-wise hashing method to approximately count triangles for each individual node in an undirected simple graph. Our method does not rely on counting triangles for each individual node. Rather, we use a carefully designed estimator to estimate the statistics from a sampled graph, which is demonstrated to be efficient and accurate.

7. CONCLUSION
Online social networks provide various ways for users to interact with other users or media content over the Internet, which bridge the online and offline worlds tightly. This provides an opportunity to researchers to leverage online user interactions so as to detect bursts that may cause impact to the offline world. We find that the emergence of bursts caused by either user-user interaction or user-content interaction are accompanied with the formation of triangles in users’ interaction networks. This finding prompts us to devise a new method for burst detection in OSNs by introducing the triadic cardinality distribution. Triadic cardinality distribution is demonstrated to be robust against common spams which makes it a more suitable indicator for detecting bursts than the volume of user activities. We design a sample-estimate solution that can efficiently estimate motif statistics in a large network.

8. REFERENCES
Figure 13: Burst detection during the 2014 Hong Kong Occupy Central movement in Twitter