Friends or Foes: Distributed and Randomized Algorithms to Determine Dishonest Recommenders in Online Social Networks

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Abstract—Viral marketing is becoming important due to the popularity of online social networks (OSNs). Companies may provide incentives (e.g., via free samples of a product) to a small group of users in an OSN, and these users provide recommendations to their friends, which eventually increases the overall sales of a given product. Nevertheless, this also opens a door for "malicious behaviors": dishonest users may intentionally give misleading recommendations to their friends so as to distort the normal sales distribution. In this paper, we propose a detection framework to identify dishonest users in OSNs. In particular, we present a set of fully distributed and randomized algorithms, and also quantify the performance of the algorithms by deriving probability of false positive, probability of false negative, and the distribution of number of detection rounds. Extensive simulations are also carried out to illustrate the impact of misleading recommendations and the effectiveness of our detection algorithms. The methodology we present here will enhance the security level of viral marketing in OSNs.

Index Terms—Dishonest Recommenders, Misbehavior Detection, Distributed Algorithms, Online Social Networks

1 INTRODUCTION

In the past few years, we have witnessed an exponential growth of user population in online social networks (OSNs). Popular OSNs such as Facebook, Twitter and Taobao [1] have attracted millions of active users. Moreover, due to the rapid development of intelligent cell phones and their integration of online social networking services [37], [38], many users have integrated these services into their daily activities, and they often share various forms of information with each other. For example, users share their opinions on purchased products with their friends, and they may also receive or even seek recommendations from their friends before doing any purchase. Therefore, when one buys a product, she may be able to influence her friends to do further purchases. This type of influence between users in OSNs is called the word-of-mouth effect, and it is also referred as social influence.

Due to the large population and the strong social influence in OSNs, companies are also adapting a new way to reach their potential customers. In particular, instead of using the conventional broadcast-oriented advertisement (e.g., through TV or newspaper), companies are now using target-oriented advertisement which takes advantage of the social influence so to attract users in OSNs to do their purchases. This new form of advertisement can be described as follows: firms first attract a small fraction of initial users in OSNs by providing free or discounted samples, then rely on the word-of-mouth effect to finally attract a large amount of buyers. As the word-of-mouth effect spreads quickly in social networks, this form of advertisement is called the viral marketing, which is a proven and effective way to increase the sales and revenue for companies [16], [19], [25], [33].

We like to emphasize that viral marketing in OSNs do exist in the real world. One particular prime example is the Taobao [1], which is one of the major operations under the Alibaba group, and it is the biggest e-commerce website in China. As of June 2013, Taobao has over 500 million registered users and 60 million of regular visitors per day. It also hosts more than 800 million types of products and represents 100 million dollars turnover per year [5], [6]. Users can buy various types of products from Taobao, and they can also run their own shops in selling products. Moreover, Taobao also developed an application addon called Friends Center on top of the website. With this application addon, users in Taobao can follow other users just like the following relationship in Twitter, hence, an OSN is formed on top of this e-commerce system. In this OSN, users can share many types of information with their friends, including the products they purchased, the shops they visited, as well as the usage experiences or opinions on products or shops. In particular, users can also forward their friends’ posts or even give comments. In addition to using the Friends Center in Taobao, one can also associate her Taobao account with her account in Sina Weibo [2], [3], which is the biggest OSN in China. According to Weibo’s prospectus, the monthly active users of Weibo reached 143.8 million in March 2014, and the daily active users also reached 66.6 million [7]. By associating Taobao
account with Sina Weibo, users can easily share their purchasing experience and ratings on products with their friends in Weibo. Based on Taobao and Weibo, many companies can easily perform target-oriented advertisement to promote their products. In fact, this type of advertisement can be easily launched in any OSN.

However, the possibility of doing target-oriented advertisement in OSNs also opens a door for malicious activities. Precisely, dishonest users in an OSN may intentionally give misleading recommendations to their neighbors, e.g., by giving high (low) rating on a low-quality (high-quality) product. To take advantage of the word-of-mouth effect, firms may also hire some users in an OSN to promote their products. In fact, this type of advertisement becomes very common in Taobao and Weibo. Worse yet, companies may even consider paying users to bad-mouth their competitors’ products. Due to the misleading recommendations given by dishonest users, even if a product is of low quality, people may still be misled to purchase it. Furthermore, products of high quality may lose out since some potential buyers are diverted to other low-quality products.

As we will show in Section 7.2 via simulation, misleading recommendations made by dishonest users indeed have a significant impact on the market share. In particular, a simple strategy of promoting one’s own product while bad-mouthing competitors’ products can greatly enhance the sales of one’s product. Furthermore, even if a product is of low quality, hiring a small percentage of users to promote it by providing misleading recommendations can severely shift the market share on various products. Therefore, it is of big significance to identify dishonest users and remove them from the networks so as to maintain the viability of viral marketing in OSNs. With respect to the normal users in OSNs, it is also of big interest to identify the dishonest users among their neighbors so as to obtain more accurate recommendations and make wiser decisions on purchasing products. Motivated by this, this paper addresses the problem of detecting dishonest recommenders in OSNs, in particular, how can a normal user discover and identify foes from a set of friends during a sequence of purchases?

However, it is not an easy task to accurately identify dishonest users in OSNs. First, an OSN usually contains millions of users, and the friendships among these users are also very complicated, which can be indicated by the high clustering coefficient of OSNs. Second, users in an OSN interact with their friends very frequently, which makes it difficult to identify dishonest users by tracing and analyzing the behaviors of all users in a centralized way. Last but not the least, in the scenario of OSNs, honest users may also have malicious behaviors unintentionally, e.g., they may simply forward the received misleading recommendations given by their dishonest neighbors without awareness. Conversely, dishonest users may also act as honest ones sometimes so as to confuse their neighbors and try to evade the detection. Therefore, the distinction between dishonest users and honest ones in terms of their behaviors becomes obscure, which finally makes the detection more challenging.

To address the problem of identifying dishonest recommenders in OSNs, this work makes the following contributions:

- We propose a fully distributed and randomized algorithm to detect dishonest recommenders in OSNs. In particular, users in an OSN can independently execute the algorithm to distinguish their dishonest neighbors from honest ones. We further exploit the distributed nature of the algorithm by integrating the detection results of neighbors so as to speed up the detection, and also extend the detection algorithm to handle network dynamics, in particular, the “user churn” in OSNs.
- We provide theoretical analysis on quantifying the performance of the detection algorithm, e.g., probability of false positive, probability of false negative, and the distribution of time rounds needed to detect dishonest users.
- We carry out extensive simulations to validate the accuracy of the performance analysis, and further validate the effectiveness of our detection algorithm using a real dataset.

The outline of this paper is as follows. In Section 2, we review related work and illustrate the difference of detecting dishonest users in OSNs from that in general recommender systems. In Section 3, we formulate the type of recommendations and the behavior of users in OSNs. In Section 4, we present the detection algorithm in detail, and also provide theoretical analysis on the performance of the algorithm. In Section 5, we develop a cooperative algorithm to speed up the detection, and in Section 6, we design a scheme to deal with the network dynamics of OSNs. We demonstrate the severe impact of misleading recommendations and validate the effectiveness of the detection algorithms via simulations in Section 7, and finally conclude the paper in Section 8.

2 RELATED WORK

A lot of studies focus on the information spreading effect in OSNs, e.g., [22], [29], and results show that OSNs are very beneficial for information spreading due to their specific natures such as high clustering coefficient. To take advantage of the easy-spreading nature and the large population of OSNs, viral marketing which is based on the word-of-mouth effect is becoming popular and has been widely studied, e.g., [16], [19], [25], [33]. In particular, because of the strong social influence in OSNs, a small fraction of initial buyers can even attract a large amount of users to finally purchase the product [29], [39]. A major portion of viral marketing research thinks viral marketing as an information diffusion process, and then study the influence maximization problem, e.g., [22], [12]. However, viral marketing in OSNs also opens a door for malicious behaviors as dishonest recommenders...
can easily inject misleading recommendations into the system so to misguide normal users’ purchases.

In the aspect of maintaining system security, some work like [34] considers to exploit the framework of trust structure [11], [23], [36]. The rough idea is to compute a trust value for every pair of nodes in distributed systems. This framework is suitable for building delegation systems and reputation systems, while it still faces a lot of challenges to address the problem of identifying dishonest users in OSNs studied in this work. First, OSNs usually contain millions of users and billions of links, the cost on computing the trust values for every pair of users must be extremely high. Second, even if the trust values for every pair of users have been computed, it still requires a mapping from the trust value to a tag indicating whether a user is dishonest or not, which is also a challenging task, especially when users have no priori information about the number of dishonest users.

With respect to malicious behavior detection, it was widely studied in wireless networks (e.g., [21], [35], [28]), P2P networks (e.g., [30], [27]), general recommender systems [8] (e.g., [13], [14], [24]), and online rating systems (e.g., [31], [17], [32]). Unlike previous works, in this paper we address the problem of malicious behavior detection in different application scenarios, online social networks. In particular, we focus on the identification of dishonest recommenders in OSNs, which is a very different problem and also brings different challenges even comparing to the problems of shill attack detection in recommender systems and review spam detection in online rating systems, both of which are considered to be more similar to the problem we considered in this paper. For example, every user in an OSN may give recommendations to her friends, while this is totally different from the case of recommender system where recommendations are made only by the system and are given to users in a centralized way. Second, users’ ratings, i.e., the recommendations, may propagate through the network for OSNs, while there is no recommendation propagation in recommender systems. And this forwarding behavior makes normal users in OSNs also have the chance of doing malicious activities, e.g., forwarding neighbors’ misleading recommendations without awareness. Last but not the least, an OSN usually has an extremely large number of nodes and links and also evolves dynamically, so a distributed detection algorithm becomes necessary with the consideration of the computation cost. However, this increases the difficulty of the detection because the detector only has the local information including her own purchasing experience and the recommendations received from her neighbors, but not the global information of the whole network as in recommender systems. In terms of the detection methodology, related work studying review spam detection usually uses machine leaning techniques, while our detection framework is based on suspicious set shrinkage with distributed iterative algorithms.

3 PROBLEM FORMULATION

In this section, we first present the model of OSNs and give the formal definitions on different types of recommendations, then we formalize the behaviors of users in OSNs on how to provide recommendations. In particular, considering that the objective of dishonest users is to promote their target products and decrease the chance of being detected, we formalize the behaviors of dishonest users into a probabilistic strategy.

3.1 Modeling on Online Social Networks

We model an OSN as an undirected graph $G = (V, E)$, where $V$ is the set of nodes in the graph and $E$ is the set of undirected edges. Each node $i \in V$ represents one user in an OSN, and each link $(i, j) \in E$ indicates the friendship between user $i$ and user $j$, i.e., user $i$ is a neighbor or friend of user $j$ and vice versa. That is, user $i$ and user $j$ can interact with each other via the link $(i, j)$, e.g., give recommendations to each other. Usually, OSNs are scale-free [39], [29] and the degrees of nodes follow power law distribution [9]. Precisely, $p(k) \propto k^{-\gamma}$, where $p(k)$ is the probability of a randomly chosen node in $G$ having degree $k$ and $\gamma$ is a constant with a typical value $2 < \gamma < 3$. We denote $N_i = \{j | (i, j) \in E\}$ as the neighboring set of user $i$ and assume that $|N_i| = N$.

3.2 Products and Recommendations

We first formalize products, and then give the definitions of different types of recommendations. We consider a set of “substitutable” products $P_1, P_2, \cdots, P_M$ that are produced by firms $F_1, F_2, \cdots, F_M$, respectively, and these firms compete in the same market. Two products are substitutable if they are compatible, e.g., polo shirts from brand X and brand Y are substitutable goods from the customers’ points of view. We characterize each product $P_j$ with two properties: (1) its sale price and (2) users’ valuations. We assume that each product $P_j$ has a unique price which is denoted as $p_j$. With respect to users’ valuations, since different users may have different ratings on a product because of their subjectivity, we denote $v_{ij}$ as the valuation of user $i$ on product $P_j$.

We categorize a product into two types according to its sale price and users’ valuations. In particular, if user $i$ thinks that a product $P_j$ is sold at the price that truly reveals its quality, then she considers this product as a trustworthy product. That is, product $P_j$ is classified as a trustworthy product by user $i$ only when $p_j = v_{ij}$. Here the equal sign means that the product is sold at a fair price from the point of view of user $i$. Conversely, if user $i$ thinks that the price of product $P_j$ does not reveal its quality, or formally, $p_j \neq v_{ij}$, then she classifies it as an untrustworthy product. Similarly, here the inequality sign just means that user $i$ thinks that $P_j$ is priced unfair, maybe much larger than its value. For example, maybe this product is of low quality or even bogus, but
it is produced by speculative and dishonest companies who always seek to maximize their profit by cheating customers. Formally, we use $T_i(P_j)$ to denote the type of product $P_j$ classified by user $i$, and we have

$$T_i(P_j) = \begin{cases} 1, & \text{if user } i \text{ considers } P_j \text{ to be trustworthy}, \\ 0, & \text{if user } i \text{ considers } P_j \text{ to be untrustworthy}. \end{cases}$$

Since products are categorized into two types, we assume that there are two types of recommendations: positive recommendations and negative recommendations, which are denoted by $R^P(P_j)$ and $R^N(P_j)$, respectively.

**Definition 1:** A positive recommendation on product $P_j$ ($R^P(P_j)$) always claims that $P_j$ is a trustworthy product regardless of its type, while a negative recommendation on $P_j$ ($R^N(P_j)$) always claims that $P_j$ is an untrustworthy product regardless of its type. Formally, we have

$$R^P(P_j) \equiv \text{"P}_j\text{ is a trustworthy product"},$$

$$R^N(P_j) \equiv \text{"P}_j\text{ is an untrustworthy product"}.$$

Note that a recommendation, either $R^P(P_j)$ or $R^N(P_j)$, does not reveal the type of product $P_j$ classified by users, so one may make positive (or negative) recommendations even if she takes the product as an untrustworthy (or a trustworthy) product. To have the notion of correctness, we further classify recommendations into correct recommendations and wrong recommendations by integrating users’ valuations.

**Definition 2:** A recommendation on product $P_j$ is correct for user $i$, which is denoted as $R^C_i(P_j)$, only when it reveals the type of $P_j$ classified by user $i$, i.e., $T_i(P_j)$, while a wrong recommendation on product $P_j$ for user $i$ ($R^W_i(P_j)$) reveals the opposite type of product $P_j$ classified by user $i$. Formally, we have

$$R^C_i(P_j) \equiv \begin{cases} R^P(P_j), & \text{if } T_i(P_j) = 1, \\ R^N(P_j), & \text{if } T_i(P_j) = 0. \end{cases}$$

$$R^W_i(P_j) \equiv \begin{cases} R^P(P_j), & \text{if } T_i(P_j) = 0, \\ R^N(P_j), & \text{if } T_i(P_j) = 1. \end{cases}$$

### 3.3 Behaviors of Users in OSNs

In this subsection, we formalize the behaviors of users in an OSN. We assume that for any user, if she buys a product, then she can evaluate the product based on her usage experience, and then categorizes it into either a trustworthy product or an untrustworthy product from her point of view.

**Behaviors of honest users.** We define honest users as the ones who will not intentionally give wrong recommendations. That is, if an honest user buys a product, since she can evaluate the product and determine its type, she always gives correct recommendations on the product to her neighbors. Precisely, she gives positive recommendations if the product is considered to be trustworthy and negative recommendations otherwise.

On the other hand, if an honest user did not buy a product, she may also give recommendations to her neighbors by simply forwarding the received recommendations from other ones. This type of forwarding behavior is quite common in OSNs. For example, in Taobao and Weibo, many users forward their friends’ posts, including their purchasing experiences and ratings on products. In a study of online social networks [4], it was found that 25% of users had forwarded an advertisement to other users in the network. Moreover, due to the anonymity of users’ identities (e.g., users usually use pseudo names to register their Weibo account), it is extremely difficult to trace the information spreading process in OSNs, and so users may simply forward any form of information without confirming its truthfulness.

In particular, a user may forward a positive (negative) recommendation given by her neighbors without validating the quality of the product. Because of this forwarding behavior, it is possible that honest users may give wrong recommendations to their neighbors. Thus, a user who gives wrong recommendations is not strictly dishonest, but only potentially dishonest. In other words, if the detector considers a product to be trustworthy and receives negative recommendations from a neighbor, she still cannot be certain that this neighbor is dishonest, mainly because it is possible that this neighbor does not intend to cheat, but is just misled by her neighbors.

**Behaviors of dishonest users.** We define dishonest users as the ones who may give wrong recommendations intentionally, e.g., give positive recommendations on an untrustworthy product. Note that dishonest users may also behave differently as they may aim for promoting different products, e.g., users who are hired by firm $F_i$ aim for promoting product $P_i$, while users who are hired by firm $F_j$ aim for promoting $P_j$. Without loss of generality, we assume that there are $m$ types of dishonest users who are hired by firms $F_1, F_2, \cdots, F_m$, and they promote products $P_1, P_2, \cdots, P_m$, respectively. Furthermore, we assume that the products promoted by dishonest users (i.e., products $P_1, P_2, \cdots, P_m$) are untrustworthy for all users. The intuition is that these products are of low quality (or even bogus) so that they can be easily identified by people. The main reason to make this assumption is that in this case dishonest users have incentives to promote these products for a larger profit, meanwhile, honest users also have incentives to detect such dishonest users so as to avoid purchasing untrustworthy products. To further illustrate this, note that when users in an OSN are attracted to buy a product promoted by dishonest users, if the product is a trustworthy one, then there is no difference for these buyers to purchase other trustworthy products instead of the one promoted by dishonest users, and so honest users have no incentive to identify the dishonest users who promote trustworthy products. In other words, promoting trustworthy products can be regarded as normal behaviors, so we only focus on the case where the promoted products are untrustworthy in this paper. However, we would like to point out that when we model the behaviors of dishonest users in the following,
we allow dishonest users to behave as honest ones and give correct recommendations.

Recall that the goal of dishonest users is to attract as many users as possible to purchase the product they promote, one simple and intuitive strategy to achieve this goal is to give positive recommendations on the product they promote and negative recommendations on all other products. On the other hand, besides attracting as many users as possible to buy their promoted product, dishonest users also hope to avoid being detected so that they can perform malicious activities for a long time. Therefore, dishonest users may also adopt a more intelligent strategy so to confuse the detector and decrease the chance of being detected. For instance, instead of always bad-mouthing other products by giving negative recommendations, they may probabilistically give correct recommendations and behave like honest users sometimes. The benefit of this probabilistic strategy is to make the detection more difficult so that dishonest users may hide in a longer time. In this paper, we allow dishonest users to adopt this intelligent strategy and use \( S_j^l \) to denote the one adopted by a type-\( l \) (\( 1 \leq l \leq m \)) dishonest user \( j \). Moreover, we allow dishonest users to be more powerful by assuming that they know honest users’ valuation on each product so that they can mislead as many users as possible. The intelligent strategy \( S_j^l \) can be formally expressed as follows.

\[
S_j^l \equiv R^P(P_l) \land \left[ \land_{n=1}^M ( \delta R^C(P_n) \lor (1-\delta) R^N(P_n) ) \right],
\]

where \( \delta \) denotes the probability of giving correct recommendations. Recall that the goal of type-\( l \) dishonest users is to attract as many users as possible to purchase product \( P_l \), while giving positive recommendations on other products (say \( P_n, n \neq l \)) goes against their objective, so we assume that dishonest users only give correct recommendations on \( P_n \) with a small probability, i.e., \( \delta \) is small. In particular, \( \delta = 0 \) implies that dishonest users always bad-mouth other products.

Note that there is a possibility that dishonest users do not adopt the probabilistic strategy as in Equation (1), while choose to promote trustworthy products over a long time just to create a good reputation, and then behave maliciously by giving misleading recommendations on a product. However, as long as the dishonest users start performing malicious activities, our detection framework still provides us with the opportunity of detecting them as we can keep executing the detection algorithm continuously. On the other hand, even if our framework may fail to detect the dishonest users if they only perform malicious activities in a very limited number of rounds, the effect of misleading recommendations given by dishonest users is also very limited, and so the corresponding miss detection error should be very small.

Another possibility we would like to point out is that multiple dishonest users may collude and a single dishonest user may also create multiple Sybil accounts to consistently promote a low-quality product. This type of collaborated malicious attack is still detectable under our framework, this is because our detection framework is fully distributed and when the detector determines whether a neighbor is dishonest or not, she only relies on her own valuation on a product and the recommendation given by this neighbor. Therefore, the possibility of a dishonest user being detected only depends on the amount of malicious activities she performs, and it is irrelevant to other users’ behaviors. However, for the cooperative detection algorithm that is developed for speeding up the detection, dishonest users may evade the detection if they collude as the detector may determine the type of a neighbor by exploiting other neighbors’ detection information, while the possibility of evading the detection depends on the parameters controlled by the detector. Hence, there is a tradeoff between detection accuracy and detection efficiency, and we will further illustrate this in Section 5.

### 3.4 Problem
In this paper, we develop distributed algorithms that can be run at any user in an OSN to identify her dishonest neighbors. Specifically, we first develop a randomized baseline algorithm which only exploits the information of the detector, see Section 4 for details. We also quantify the performance of the algorithm via theoretical analysis. Then we propose a cooperative algorithm which further takes advantage of the detection results of the detector’s neighbors so as to speed up the detection, see Section 5 for details. After that, we further extend the algorithm to deal with network dynamics of OSNs, i.e., user churn, in Section 6.

### 4 Baseline Detection Algorithm
In this section, we first illustrate the rough idea of the detection framework, and then present the detection algorithm in detail. We also quantify various performance measures of the algorithm.

#### 4.1 General Detection Framework
Our detection algorithm is fully distributed, and so users can independently execute it to identify dishonest users among their neighbors. Without loss of generality, we only focus on one particular user, say user \( i \), and call her the detector. That is, we present the algorithm from the perspective of user \( i \) and discuss how to detect her dishonest neighbors. For ease of presentation, we simply call a product as a trustworthy (or untrustworthy) product if the detector considers it to be trustworthy (or untrustworthy).

Note that even if users’ subjectivity creates different preferences on different products, we assume that the detector and her neighbors have a consistent valuation on most products. This assumption is reasonable, especially for the cases where the quality of products can be easily identified, and its rationality can be further justified as
follows. First, users in an OSN prefer to have friends with others who share similar interests and tastes. Hence users in an OSN are similar to their neighbors [15] and so they have a consistent valuation with their neighbors on many products. Secondly, “wisdom of the crowd” is considered to be the basis of online rating systems like Amazon and Epinions, and it is also widely used by people in their daily lives, so it is reasonable to assume that most products have intrinsic quality so that the detector and her neighbors will have a consistent rating.

Note that the above assumption allows users who are not friends with each other to have very different valuations on the same product, and it also allows the detector and her neighbors to have different valuations on some products. In fact, if an honest neighbor has a different rating on a product, then from the detector’s point of view, it is just equivalent to the case where this honest neighbor is misled by dishonest users and so gives a wrong recommendation. Another issue we would like to point out is that even if the above assumption does not hold, e.g., if the detector and her neighbors have different ratings on all products, our detection framework still provides a significant step toward identifying dishonest behavior in OSNs’ advertisements. This is because if a neighbor has different valuations on all products from the detector, then our detection framework will take this neighbor as “misleading” no matter she intends to cheat or not. This is acceptable as users always prefer neighbors to have the similar taste (or preference) with them so that they can purchase a product they really like if they take their neighbors’ recommendations.

We model the purchase experience of detector $i$ as a discrete time process. Particularly, we take the duration between two continuous purchases made by detector $i$ as one round, and time proceeds in rounds $t = 1, 2, \ldots$. That is, round $t$ is defined as the duration from the time right before the $t^{th}$ purchase instance to the time right before the $(t+1)^{th}$ purchase instance. Based on this definition, detector $i$ purchases only one product at each round, while she may receive various recommendations on the product from her neighbors, e.g., some neighbors may give her positive recommendations and others may give her negative recommendations.

The general idea of our detection framework can be illustrated as in Figure 1. Initially, detector $i$ is conservative and considers all her neighbors as potentially dishonest users. We use $S_i(0)$ to denote the set of potentially dishonest neighbors of detector $i$ until round $t$, which is termed as the suspicious set, and we have $S_i(0) = \mathcal{N}_i$. As time proceeds, detector $i$ differentiates her neighbors based on their behaviors in each round, and shrinks the suspicious set by removing her trusted neighbors which are classified as honest users. After sufficient number of rounds, one can expect that all honest neighbors are removed from the suspicious set and only dishonest neighbors left. Therefore, after $t$ rounds, detector $i$ takes a neighbor as dishonest if and only if this neighbor belongs to the suspicious set $S_i(t)$.

![Fig. 1: General detection process via suspicious set shrinkage.](image-url)

4.2 Operations in One Round

In this subsection, we describe the detailed operations of shrinking the suspicious set in only one round, say round $t$. Note that detector $i$ buys a product at round $t$, which we denote as $P_{ji}$ ($j_i \in \{1, 2, \ldots, M\}$), so she can valuate the product and determine its type $T_i(P_{ji})$ from her point of view. Moreover, she can further categorize the received recommendations (that are either positive or negative) into correct recommendations and wrong recommendations based on her valuation on the product, and so she can differentiate her neighbors according to their recommendations. Specifically, we define $N_i^C(t)$ as the set of neighbors whose recommendations given at round $t$ are classified as correct by detector $i$. Accordingly, we denote $N_i^W(t)$ and $N_i^N(t)$ as the set of neighbors who give detector $i$ wrong recommendations and no recommendation at round $t$, respectively. We have $N_i(t) = N_i^C(t) \cup N_i^W(t) \cup N_i^N(t)$.

Recall that a product is either trustworthy or untrustworthy, so detector $i$ faces two cases at round $t$: (1) the purchased product $P_{ji}$ is a trustworthy product, i.e., $T_i(P_{ji}) = 0$, and (2) the purchased product $P_{ji}$ is a trustworthy product, i.e., $T_i(P_{ji}) = 1$. In the following, we illustrate on how to shrink the suspicious set in the above two cases.

In the first case, a neighbor who gives correct recommendations can not be certainly identified as honest, mainly because a dishonest neighbor may also give correct recommendations. For example, a type-1 ($l \neq j_i$) dishonest user may give negative recommendations on product $P_{ji}$ based on the intelligent strategy, and this recommendation will be classified as correct as the detector values product $P_{ji}$ as an trustworthy product. Therefore, detector $i$ is not able to differentiate honest neighbors from dishonest ones if $T_i(P_{ji}) = 0$. We adopt a conservative policy by keeping the suspicious set unchanged, i.e., $S_i(t) = S_i(t-1)$.

In the second case, since detector $i$ evaluates product $P_{ji}$ as a trustworthy product, $P_{ji}$ cannot be a product promoted by dishonest users, and we have $P_{ji} \in \{P_{m+1}, \ldots, P_{M}\}$. In this case, even if a dishonest user may give correct recommendations on product $P_{ji}$ based on the intelligent strategy, the corresponding probability $\delta$ is considered to be small, and so a dishonest user should belong to the set $N_i^W(t)$ with high probability. Note that it is also possible that dishonest users do not make any recommendation at round $t$, so dishonest users can be in
Algorithm 1 Randomized Detection Algorithm at Round \( t \) for Detector \( i \):

1. Estimate the type of the purchased product \( P_{ij} \).
2. Differentiate neighbors by determining \( N^W_i(t) \), \( N^N_i(t) \), and \( N^V_i(t) \).
3. Let \( D(t) \leftarrow N^W_i(t) \cap N^N_i(t) \).
4. if \( T_i(P_{ij}) \) = 1 then
5. with probability \( p \), \( S_i(t) \leftarrow S_i(t-1) \cap D(t) \);
6. with probability \( 1-p \), \( S_i(t) \leftarrow S_i(t-1) \);
7. else
8. \( S_i(t) \leftarrow S_i(t-1) \);
9. end if

To further illustrate the detection process in Algorithm 1, we consider Figure 2 as an example to show the operations at round \( t \). User \( i \) have seven neighbors that are labeled from \( a \) to \( g \). Assume that two of them are dishonest (i.e., user \( a \) and \( b \)). Suppose that neighbors \( a, b, c, d, e \) are in the suspicious set before round \( t \), i.e., \( S_i(t-1) = \{a, b, c, d\} \). We use dashed cycles to denote suspicious users in Figure 2. If user \( i \) buys a trustworthy product at round \( t \), and only neighbors \( e, f \) give her correct recommendations, then user \( i \) can be certain that neighbor \( e \) is honest with a high probability and it can be removed from the suspicious set. Therefore, according to Algorithm 1, the suspicious set shrinks with probability \( p \), and if this probabilistic even happens, then we have \( S_i(t) = \{a, b, c\} \) as shown on the right hand side of Figure 2.

To derive the above three performance measures for Algorithm 1, note that the suspicious set shrinks at twofold. First, the size of an OSN is usually very large, e.g., it may contain millions of nodes and billions of links, so a distributed algorithm becomes necessary so as to make the computation feasible. Second, an OSN itself is fully distributed, in particular, a user in an OSN only receives information, e.g., recommendations on products, from her direct neighbors, and so she only needs to care about the honesty of her neighbors so as to make the received recommendations more accurate. Therefore, a fully distributed detection algorithm is indeed necessary for the application we consider.

4.3 Performance Evaluation

To characterize the performance of the detection algorithm, we define three performance measures: (1) probability of false negative which is denoted as \( P_{fn}(t) \), (2) probability of false positive which is denoted as \( P_{fp}(t) \), and (3) the number of rounds needed to shrink the suspicious set until it only contains dishonest users, which is denoted by a random variable \( R \). Specifically, \( P_{fn}(t) \) characterizes the probability that a dishonest user is wrongly regarded as an honest one after \( t \) rounds, and \( P_{fp}(t) \) characterizes the error that an honest user is wrongly regarded as a dishonest one after \( t \) rounds. Recall that detector \( i \) takes a neighbor \( j \in N_i \) as dishonest if and only if this neighbor belongs to the suspicious set (i.e., \( j \in S_i(t) \)), so we define \( P_{fn}(t) \) as the probability that a dishonest neighbor of detector \( i \) is not in \( S_i(t) \) after \( t \) rounds. Formally, we have

\[
P_{fn}(t) = \frac{\# \text{ of dishonest neighbors of } i \text{ that are not in } S_i(t)}{\# \text{ of dishonest neighbors of detector } i}.
\]

On the other hand, since all neighbors of detector \( i \) are initially included in the suspicious set (i.e., \( S_i(0) = N_i \)), an honest user is wrongly regarded as a dishonest one only if she still remains in the suspicious set after \( t \) rounds. Thus, we define \( P_{fp}(t) \) as the probability of an honest user not being removed from the suspicious set after \( t \) rounds. Formally, we have

\[
P_{fp}(t) = \frac{\# \text{ of honest neighbors of } i \text{ that are in } S_i(t)}{\# \text{ of honest neighbors of detector } i}.
\]
round $t$ only when detector $i$ valuates her purchased product as a trustworthy product and this round is further used for detection with probability $p$. We call such a round a detectable round and use a 0-1 random variable $d(t)$ as an indicator, where $d(t) = 1$ means that round $t$ is detectable and 0 otherwise. In addition to the indicator $d(t)$, detector $i$ also obtains the set $D(t)$ to which dishonest users may belong at round $t$. Therefore, we use a tuple $(d(t), D(t))$ to denote the information that detector $i$ obtains at round $t$, and the set of all tuples until round $t$ constitute the detection history, which we denote as $H(t)$. Formally, we have

$$ H(t) = \{(d(1), D(1)), (d(2), D(2)), \ldots, (d(t), D(t))\}. $$

Based on the detection history $H(t)$, the performance measures of $P_{fn}(t)$, $P_{fp}(t)$ and the distribution of $R$ for Algorithm 1 can be derived as in Theorem 1.

**Theorem 1:** After running Algorithm 1 for $t$ rounds, probability of false negative and probability of false positive are derived in Equation (4) and Equation (5), respectively.

$$ P_{fn}(t) = 1 - (1 - \delta)\sum_{r=1}^{t} d(r), \quad (4) $$

$$ P_{fp}(t) \approx \prod_{\tau=1}^{t} \frac{|D(\tau - 1) \cap D(\tau)|}{|D(\tau - 1)|}, \quad (5) $$

where $D(0) = N_i$ and $D(\tau)$ is set as $D(\tau - 1)$ if $d(\tau) = 0$. The number of rounds needed for detection until the suspicious set only contains dishonest users follows the distribution of

$$ P(R=r) = \sum_{d=1}^{r} \binom{r}{d}(p_d)^d(1-p_d)^{r-d} \times \left[\left[1 - (1 - p_{hc})^d \right]^{N-k} - [1 - (1 - p_{hc})^{d-1}]^{N-k}\right]. \quad (6) $$

where $p_{hc}$ is the average probability of an honest user giving correct recommendations at each round and $p_d$ is the probability of a round being detectable, which can be estimated by Equation (9) and Equation (10) in the Appendix, respectively.

**Proof:** Please refer to the Appendix.

Since probability of false positive $P_{fp}(t)$ is critical to design the complete detection algorithm (see Section 4.4), we use an example to further illustrate its derivation. Note that a user in an OSN usually has a large number of friends, so we let detector $i$ have 100 neighbors labeled from 1 to 100. Among these 100 neighbors, we assume that the last two are dishonest, whose labels are 99 and 100. Before starting the detection algorithm, we initialize $D(0)$ as $N_i$ and let $P_{fp}(0) = 1$.

In the first detection round, suppose that user $i$ buys a trustworthy product and further takes this round as detectable. Besides, suppose that only neighbor 1 and neighbor 2 give her correct recommendations, i.e., $D(1) = \{3, 4, \cdots, 100\}$, then we have $S_i(1) = \{3, 4, \cdots, 100\}$. Based on Equation (5), the probability of false positive can be derived as

$$ P_{fp}(1) = P_{fp}(0) \times \frac{|D(0) \cap D(1)|}{|D(0)|} = 0.98. $$

Note that according to the definition in Equation (3), the accurate value of probability of false positive is $\frac{2}{99}$, which is a little bit smaller than the result derived by Theorem 1. In fact, Theorem 1 provides a good approximation when the number of neighbors is large and the number of dishonest users among them is small, which is the common case for OSNs as users often tend to have a lot of friends and a company can only control a small number of users to promote its product.

Now let us consider the second detection round. Suppose that the event with probability $p$ does not happen. That is, this round is not detectable. So we set $D(2) = D(1)$, and the suspicious set remains the same, i.e., $S_i(2) = S_i(1) = \{3, 4, \cdots, 100\}$. The probability of false positive is still

$$ P_{fp}(2) = 0.98. $$

We further examine one more round. Suppose that the third round is detectable and neighbor 1 to neighbor 4 give user $i$ correct recommendations, i.e., $D(3) = \{5, \cdots, 100\}$. Based on Algorithm 1, we have $S_i(3) = S_i(2) \cap D(3) = \{5, \cdots, 100\}$. The probability of false positive can be derived as

$$ P_{fp}(3) = P_{fp}(2) \times \frac{|D(2) \cap D(3)|}{|D(2)|} = 0.96. $$

Note that according to the definition in Equation (3), the accurate value after round $t$ is $\frac{4}{95} = 0.959$. Based on Theorem 1, we see that $P_{fp}(t) \to 0$, and this implies that all honest users will be removed from the suspicious set eventually. However, $P_{fn}(t)$ does not converge to zero, which implies that dishonest users may evade the detection. Fortunately, as long as $P_{fn}(t)$ is not too large when $P_{fp}(t)$ converges to zero, one can still effectively identify all dishonest users (as we will show in Section 7) by executing the detection process multiple times. On the other hand, the expectation of $R$ quantifies the efficiency of the detection algorithm, in particular, it indicates how long a detector needs to identify her dishonest neighbors on average. Note that the detection algorithm itself does not rely on the derivation of this performance measure, and it is just used for studying the detection efficiency of the algorithm.

### 4.4 Complete Detection Algorithm

In Section 4.2, we present a partial detection algorithm which describes the operations in a particular round $t$. In this subsection, we present the corresponding complete algorithm which describes how to shrink the suspicious set until dishonest users can be identified. To achieve this, we have to determine the termination condition when repeating the partial algorithm round by round. Observe that after executing the detection algorithm for $t$ rounds, only users in the suspicious set $S_i(t)$ are taken as dishonest ones. Intuitively, to avoid a big detection error, the detection process can only be terminated when users in $S_i(t)$ are really dishonest with high probability.
Based on the definition of probability of false positive $P_{fp}(t)$, it is sufficient to terminate the algorithm when $P_{fp}(t)$ is lower than a predefined small threshold $P_{fp}^*$. In other words, as long as probability of false positive is small enough, we can guarantee that all users in the suspicious set are really dishonest with high probability. Based on the above illustration, the complete detection algorithm can be stated as follows.

Algorithm 2 Complete Detection Algorithm

1: $t \leftarrow 0$
2: $S_i(0) \leftarrow N_i$
3: repeat
4: $t \leftarrow t + 1$
5: Derive the suspicious set $S_i(t)$ at round $t$ by executing Algorithm 1;
6: Update probability of false positive $P_{fp}(t)$;
7: until $P_{fp}(t) \leq P_{fp}^*$
8: Take users in $S_i(t)$ as dishonest and blacklist them;

5 Cooperative Algorithm to Speed up the Detection

In the last section, we propose a distributed and randomized algorithm that only exploits the detector’s local information. By running this algorithm, honest users can detect their dishonest neighbors simultaneously and independently. That is, each user in an OSN maintains her own suspicious set containing her potentially dishonest neighbors. Since users in an OSN interact with each other frequently, they can also share their detection results, e.g., their suspicious sets. By doing this, a detector can further exploit her neighbors’ detection history to speed up her own detection, and we term this scenario as cooperative detection.

We still focus on a particular detector, say user $i$, and use $S_i(t)$ to denote her suspicious set. At round $t$, user $i$ may shrink her suspicious set based on her purchasing experience and her received recommendations, and she may also request the detection results of her neighbors. In particular, we assume that detector $i$ can obtain two sets from each neighbor $j$ at round $t$: the neighboring set and the suspicious set of neighbor $j$, which we denote as $N_j$ and $S_j(t)$, respectively.

To exploit neighbors’ detection results, at round $t$, detector $i$ first shrinks her own suspicious set according to Algorithm 1, and we call this step as the independent detection step. After that, detector $i$ further shrinks her suspicious set by exploiting the information received from her neighbors (i.e., $(N_j, S_j(t)), j \in N_i$), and we term this step as the cooperative detection step. Since detector $i$ may have different degrees of trust on her neighbors, we use $w_{ij}(t) (0 \leq w_{ij}(t) \leq 1)$ to denote the weight of trust of user $i$ on neighbor $j$ at round $t$. That is, user $i$ only exploits the detection results of neighbor $j$ with probability $w_{ij}(t)$ at round $t$. Intuitively, $w_{ij}(t) = 1$ implies that user $i$ fully trusts neighbor $j$, while $w_{ij}(t) = 0$ means that user $i$ does not trust $j$ at all. The cooperative detection algorithm for user $i$ at round $t$ is stated in Algorithm 3.

Algorithm 3 Cooperative Detection Algorithm at Round $t$ for Detector $i$

1: Derive the suspicious set $S_i(t)$ based on local information (i.e., using Algorithm 1);
2: Exchange detection results with neighbors;
3: for each neighbor $j \in N_i$ do
4: with probability $w_{ij}(t)$: $S_i(t) \leftarrow S_i(t) \cup (N_j \setminus S_j(t));$
5: with probability $1 - w_{ij}(t)$: $S_i(t) \leftarrow S_i(t);$ 
6: end for

We take Figure 3 as an example to further illustrate the operations at the cooperative detection step (i.e., Line 3-6 in Algorithm 3). Since user $i$ first shrinks her suspicious set by using Algorithm 1, we still use the setting in Figure 2 where $S_i(t)$ shrinks to $\{a, b, c\}$ after the first step. Now to further exploit neighbors’ detection results to shrink $S_i(t)$, suppose that only user $c$ is a neighbor of user $d$, and it has already been removed from user $d$’s suspicious set. That is, $c \in N_d$ and $c \notin S_d$. If user $i$ fully trusts neighbor $d$ (i.e., $w_{id}(t) = 1$), then user $i$ can be certain that neighbor $c$ is honest as $c$ is identified as honest by neighbor $d$. Thus, user $i$ can further shrink her suspicious set, and we have $S_i(t) = \{a, b\}$ as shown on the right hand side of Figure 3.

Fig. 3: An example illustrating Algorithm 3.

To implement Algorithm 3, we need to set the weights of trust on different neighbors, i.e., $w_{ij}(t)$. One simple strategy is only trusting the neighbors that are not in the suspicious set as users in the suspicious set are potentially dishonest. Mathematically, we can express this strategy as follows.

$$w_{ij}(t) = \begin{cases} 0, & \text{if } j \in S_i(t), \\ 1, & \text{otherwise}. \end{cases}$$ (7)

Note that $w_{ij}(t)$ is a tunable parameter for detector $i$, and it affects the shrinking rate of the suspicious set of detector $i$. On the other hand, since detector $i$ may further shrink her suspicious set by exploiting her neighbors’ detection results, dishonest users may evade the detection if they collude, while the possibility also depends on the parameter $w_{ij}(t)$. In fact, there is a tradeoff
between detection accuracy and efficiency when choosing this parameter. Specifically, larger $w_{ij}(t)$’s imply that detector $i$ is more aggressive to further exploit her neighbors’ detection results, and so the detection rate should be larger, while the risk of dishonest users evading the detection also becomes larger.

Again, Algorithm 3 is only a partial algorithm that describes the operation at round $t$. To develop the complete version of the cooperative detection algorithm, we can still use the idea in Section 4.4 to set the termination condition. That is, we keep running Algorithm 3 until probability of false positive is less than a predefined threshold $Pfp^*$. To achieve this, we have to derive the probability of false positive $Pfp(t)$ for Algorithm 3, and the result is stated in Theorem 2.

Theorem 2: After running Algorithm 3 for $t$ rounds, probability of false positive can be derived as follows.

$$Pfp(t) \approx \frac{Pfp(t-1)^{2|D(t-1)|2|D(t)|}}{2^{(t-1)}} N - |C(t)|$$

where $Pfp(0) = 1$ and $C(t)$ denotes the set of neighbors that are removed from the suspicious set in the cooperative detection step at round $t$.

Proof: Please refer to the Appendix.

6 Algorithm Dealing with User Churn

In previous sections, we proposed a randomized detection algorithm and also discussed about how to speed up the detection. These algorithms are designed based on the assumption that the underlying network is static, i.e., the friendships between users are fixed and do not change during the detection. However, an online social network usually evolves dynamically, in particular, new users may join in the network and existing users may change their friendships or even leave the network by deleting their profiles [18], [26], [40]. Taking the network dynamics into consideration, for detector $i$, new users may become her friends and existing friends may also disconnect with her at some time. We call these behaviors as user churn. Note that even if users may leave the network and reconnect after some time, while they may not be able to recover the past friendships as establishing links or friendships usually requires the confirmation of other users in OSNs. In this section, we extend our detection algorithm to address the problem of user churn in OSNs.

We still focus on a particular detector, say user $i$. At each round, we first employ previous algorithms, e.g., Algorithm 1 or Algorithm 3, to shrink the suspicious set. After that, we do the following checks: (1) whether there are new users becoming the neighbors of detector $i$, and (2) whether some existing neighbors of detector $i$ disconnect with her. In particular, if new neighbors come in, we add them into the neighboring set $N_i$ and the suspicious set $S_i(t)$. In other words, we are conservative to take new users as potentially dishonest. For ease of presentation, we use $N_u(t)$ to denote the set of new users that become the neighbors of detector $i$ at round $t$. On the other hand, if some existing neighbors disconnect with detector $i$ at round $t$, we simply remove them from both the neighboring set $N_i$ and the suspicious set $S_i(t)$. We use $L(t)$ to denote the set of neighbors that leave detector $i$ at round $t$, and use $L_S(t)$ to denote the set of users that are in the suspicious set $S_i(t)$ and leave detector $i$ at round $t$, i.e., $L_S(t) = S_i(t) \cap L(t)$. Now we present the detailed detection algorithm at round $t$ in Algorithm 4. Note that if Algorithm 3 is used to shrink the suspicious set in Algorithm 4, then cooperative detection is used to speed up the detection.

Algorithm 4 Dealing with User Churn at Round $t$

1: Derive the suspicious set $S_i(t)$ (by executing Algorithm 1 or Algorithm 3);
2: Derive the set $N_u(t)$ and $L(t)$;
3: $S_i(t) \leftarrow (S_i(t) \cup N_u(t)) \setminus L(t)$;
4: $N_i \leftarrow (N_i \cup N_u(t)) \setminus L(t)$;

Let us use an example to illustrate the operations in Algorithm 4 and it is shown in Figure 4. Since the suspicious set first shrinks by using Algorithm 1 or Algorithm 3, which has been illustrated before. Here we only show the step dealing with user churn (i.e., Line 2-4). Suppose that at round $t$, user $i$ disconnects with neighbor $b$ (i.e., $L(t) = \{b\}$), and initiates a connection with a new user that is labeled as $h$ (i.e., $N_u(t) = \{h\}$), then user $i$ can safely remove $b$ from the suspicious set as she does not care user $b$ any more, while she has no priori information about the type of the new user $h$, so she is conservative and add user $h$ into the suspicious set. Thus, we have $S_i(t) = \{a, h\}$ as shown in Figure 4.

![Example illustrating Algorithm 4](image)

Fig. 4: An example illustrating Algorithm 4.

The complete algorithm can also be developed by keeping running the detection process until probability of false positive is smaller than a predefined threshold $Pfp^*$. Thus, we have to derive the probability of false positive $Pfp(t)$ for Algorithm 4, and the result is stated in Theorem 3.

Theorem 3: After running Algorithm 4 for $t$ rounds, probability of false positive can be derived as follows.

$$Pfp(t) \approx \frac{Pfp(t-1)^{2|D(t-1)|2|D(t)|}}{2^{(t-1)}} N - |C(t)| - |N_u(t)| - |L_S(t)|$$

\[ N(t) \]
where \( P_f(0) = 1 \) and \( N(t) \) denotes the number of neighbors after round \( t \).

**Proof:** Please refer to the Appendix.

### 7 Simulation and Model Validation

Our model aims to detect dishonest users who intentionally give wrong recommendations in OSNs. Since each user in an OSN performs her own activities continuously, e.g., purchasing a product, giving recommendations to her neighbors, and making decisions on which product to purchase, the network evolves dynamically. Therefore, we first synthesize a dynamically evolving social network to emulate users’ behaviors, then we show the impact of misleading recommendations and validate the analysis of our detection algorithm based on the synthetic network. We also validate the effectiveness of our detection algorithm using a real dataset drawn from an online rating network.

#### 7.1 Synthesizing A Dynamically Evolving OSN

In this subsection, we synthesize a dynamic OSN to simulate the behaviors of users in the network. To achieve this, we make assumptions on (1) how users make recommendations to their neighbors, (2) how users make decisions on purchasing which product, and (3) how fast the recommendations spread.

First, there are two types of users in the network: honest users and dishonest users. Dishonest users adopt the intelligent strategy to make recommendations. For an honest user, if she buys a product, she gives correct recommendations to her friends based on her valuation on the product. On the other hand, even if an honest user does not buy a product, she still gives recommendations based on her received recommendations. We adopt the majority rule in this case. That is, if more than half of her neighbors give positive (negative) recommendations to her, then she gives positive (negative) recommendations to others. Otherwise, she does not give any recommendation. In the simulation, we let all honest users have the same valuation on each product, and so we randomly choose an honest user as the detector in each simulation.

Second, to simulate the behaviors of users on deciding to purchase which product, we assume that an honest user buys the product with the maximum number of effective recommendations that is defined as the number of positive recommendations subtracting the number of negative recommendations. The rationale is that one buys a product that receives high ratings as many as possible and low ratings as few as possible.

Last, we assume that the spreading rate of recommendations is much higher than the purchasing rate. In other words, when one gives a positive (negative) recommendation on a particular product to her neighbors, her neighbors update their states accordingly, i.e., update the number of received positive (negative) recommendations. If the corresponding numbers satisfy the majority rule, then they further make recommendations on this product, and this process continues until no one in the system can make a recommendation according to the majority rule. Moreover, the whole process finishes before the next purchase instance made by any user in the network.

To model the evolution of the network, we assume that it starts from the “uniform” state in which all products have the same market share. During one detection round, \( 10\%|V| \) purchase instances happen, where \( |V| \) is the total number of users in the network, i.e., between two successive purchases of detector \( i \), \( 10\%|V| \) purchases are made by other users in the network. Note that the assumptions we make in this subsection are only for the simulation purpose, and our detection algorithms do not require these assumptions.

#### 7.2 Impact of Misleading Recommendations

In this subsection, we show the impact of misleading recommendations using the synthetic network. We employ the GLP model proposed in [10] that is based on preferential attachment [9] to generate a scale-free graph with power law degree distribution and high clustering coefficient. We generate a graph with around 8,000 nodes and 70,000 edges, whose clustering coefficient is around 0.3. We assume that initially no product has been purchased, and consider 10,000 purchase instances in the simulation. For each purchase instance, one user purchases and she buys the product with the maximum number of effective recommendations. After that, she gives a recommendation on the product to her friends. The recommendation will spread throughout the network until no one can make a recommendation according to the majority rule. We assume that there are five products, \( P_1, \ldots, P_5 \), and dishonest users aim to promote product \( P_1 \), which is an untrustworthy product, while the rest are trustworthy products. Our objective is to measure the fraction of purchases of each product out of the total 10,000 purchases. We run the simulation multiple times and take the average value.

![Fig. 5: Impact of misleading recommendations on the market share distribution: dishonest users aim to promote an untrustworthy product \( P_1 \).](image)
The simulation results are shown in Figure 5. First, we can see that if no dishonest user exists in the network to give misleading recommendations, the untrustworthy product $P_1$ is purchased with only a small probability. The reason why the probability is non-zero is that if a user does not receive any recommendation, she simply makes a random choice over the five products to make a purchase. However, if we randomly set 5% of users as dishonest and let them adopt the intelligent strategy to promote $P_1$ by setting $\delta = 0$, then even if $P_1$ is an untrustworthy product, it is still purchased with probability around 0.15. In other words, many users in the network are misled by these dishonest users to purchase $P_1$. In summary, the existence of dishonest users who intentionally give misleading recommendations can severely distort the market share distribution.

### 7.3 Analysis Validation via A Synthetical OSN

In this subsection, we synthesize a dynamically evolving network based on the description in Section 7.1, and then validate our analysis on the performance of the detection algorithms. In the simulation, we randomly select 5% of users as dishonest users, and let them adopt the intelligent strategy. We also randomly choose an honest user who has dishonest neighbors and take her as the detector. We carry out the simulation many times and take the average value as the simulation results.

![Fig. 6: Probability of false negative and probability of false positive for the cooperative algorithm (Algorithm 1) where $\delta = 0.1$ and $p = 0.8$.](image)

Now we focus on the detection algorithm dealing with user churn, i.e., Algorithm 4, and the results are shown in Figure 8. In the figure, one group of curves corresponds to the case where cooperative algorithm is employed, i.e., using Algorithm 3 to derive the suspicious set in the first step of Algorithm 4, the other group corresponds to the case where cooperative detection is not used, i.e., using Algorithm 1 to derive the suspicious set in the first step. To simulate user churn, we add a new neighbor to the detector with probability 0.3 in each round. Simulation results show that probability of false positive goes to zero eventually, which implies that users in the suspicious set must be dishonest with high probability after sufficient number of rounds. At last, we also observe the speedup of the detection for the cooperative algorithm.

Let us look at the distribution of number of detection rounds for the randomized detection algorithm, i.e., Algorithm 1. Results are shown in Figure 9. The horizontal
axis is the number of rounds needed for the detection, and the vertical axis is the probability mass function. We can see that even if the probability mass function is not accurately quantified, the expected number of rounds, \( E[R] \), is still well approximated. The deviation of the probability mass function can be explained as follows. First, the probability of an honest user giving correct recommendations is not a constant at each round, e.g., as more users purchase a product, the probability of giving correct recommendations also increases since more users can have their own valuations. Therefore, there must be an approximation error when we use a constant parameter, say \( p_{hc} \), to approximate it. Second, since the performance measure is quantified in a probabilistic way, it is required to run the simulation many times so as to match with the theoretic results. However, running the simulation too many times takes a lot of time because of the large graph size. To balance the tradeoff, we only run the simulation 1000 times, and the inadequate number of simulation times also contributes to the approximation error. However, since the detection algorithm does not require the accurate quantification of the distribution of \( R \), it is still effective to employ the algorithm to identify dishonest users even if an approximation error exists.

### 7.4 Evaluation on Real Data

As we stated in Section 1, the problem we considered in this paper is an abstraction of viral marketing problems in OSNs, and so there is no publicly available dataset that is drawn from an OSN specialized for viral marketing. Therefore, to further validate the effectiveness of our detection algorithm, we consider a real dataset from a social rating network, where users share their ratings on movies and also establish friendships with others. In the following, we first describe the dataset, then illustrate on how to implement our detection algorithm, and finally show the results.

**Dataset:** We use the Flixster dataset which is drawn from a social network where users share their ratings on movies with their friends [20]. The underlying social network contains around 1M users and 26.7M social relations. Users can give ratings in the range \([0.5, 5]\) with step size 0.5, and there are 8.2M ratings in this dataset. Since we classify products into two types in this work, i.e., trustworthy products and untrustworthy products, to drive evaluations using the Flixster dataset, we map each rating to a binary value (i.e., either 0 or 1) by splitting from the middle of the range (i.e., 2.5). That is, we take the ratings that are greater than 2.5 as high ratings, and consider others as low ratings. By analyzing this dataset, we find that for around 90% of movies, more than 75% of users have a consistent valuation. This also confirms the assumptions we make in our framework.

**Algorithm Implementation:** Since our detection algorithm is fully distributed and can be executed by any user. To select a detector, we randomly choose a user who has a large number of friends and also gives a lot of ratings. In particular, the detector chosen in this evaluation has around 900 friends and gives around 200 ratings. Among the 900 friends, around 700 of them have only one rating or even no rating on all of the movies the detector rated, so we ignore them in the evaluation. Since all users in this dataset are honest, to emulate malicious activities, we randomly set 10% of the detector’s neighbors as dishonest users and let them promote one particular movie. In particular, we modify the ratings given by these dishonest users based on the intelligent strategy formalized in Equation (1). We run the randomized detection algorithm (i.e., Algorithm 1) at the detector, and measure the probability of false negative and the probability of false positive based on the definitions in Equations (2)-(3) so as to validate the effectiveness of the detection algorithm.

**Detection Results:** The results of probability of false positive \( P_{fp}(t) \) and probability of false negative \( P_{fn}(t) \) are shown in Figure 10. We can see that probability of false positive continues to decrease as the algorithm executes for more and more rounds, and finally falls below a small probability. This implies that most honest users can be successfully removed from the suspicious set. On the other hand, probability of false negative also increases, which indicates the possibility of miss
detected. However, we can see that when probability of false positive drops below 0.1, probability of false negative only increases to 0.3. This shows the effectiveness of the detection algorithm. In particular, more than 70% of dishonest users can be accurately identified in one execution of the algorithm, and so we can keep executing the algorithm for multiple times so to identify all dishonest users. Another important point we like to stress is that the number of detection rounds in this evaluation is not small, e.g., probability of false positive only decreases to 0.3 after 50 rounds. The main reason is that in this dataset, most users only give very few ratings, and so many neighbors do not give any rating in most of the detection rounds, which makes them remain in the suspicious set for a long time.

**Fig. 10:** Probability of false positive and probability of false negative of the randomized detection algorithm on real dataset.

### 8 Conclusion

In this paper, we develop a set of fully distributed and randomized detection algorithms based on the idea of shrinking suspicious set so to identify dishonest users in OSNs. We formalize the behaviors of dishonest users wherein they can probabilistically bad-mouth other products while give positive recommendations on the product they aim to promote. Our detection algorithms allow users to independently perform the detection so as to discover their dishonest neighbors. We provide mathematical analysis on quantifying the effectiveness and efficiency of the detection algorithms. We also propose a cooperative scheme to speed up the detection, as well as an algorithm to handle network dynamics, i.e., “user churn” in OSNs. Via simulations, we first show that the market share distribution may be severely distorted by misleading recommendations given by a small fraction of dishonest users, and then validate the effectiveness and efficiency of our detection algorithms. The detection framework in this paper can be viewed as a valuable tool to maintain the viability of viral marketing in OSNs.

### Appendix

**Proof of Theorem 1 in Section 4.3**

We first focus on probability of false negative \( P_{fn}(t) \). Note that \( S_i(t) \) only shrinks in detectable rounds, so we have

\[
P_{fn}(t) = P\{\text{a dishonest user } j \text{ is considered to be honest}
= 1 - P\{\text{is not removed from the suspicious set in all detectable rounds}
= 1 - \prod_{\tau=1}^{t} P\{j \in D(\tau)\}.
\]

To compute the probability that a dishonest user stays in \( D(\tau) \) in a detectable round \( \tau \), observe that the product detector \( i \) purchases at this round must be a trustworthy product which is not promoted by any dishonest user. We assume that dishonest users give recommendations at every round so as to attract as many buyers as possible. Based on the intelligent strategy, a dishonest user gives correct recommendations on this product with probability \( \delta \), and this recommendation is also correct for detector \( i \) as we assume that dishonest users have the same valuation with majority users, so we have \( P\{j \in D(\tau)\} = 1 - \delta \), and probability of false negative is

\[
P_{fn}(t) = 1 - \prod_{\tau=1}^{t} (1 - \delta) = 1 - (1 - \delta)^{\sum_{\tau=1}^{t} d(\tau)}.
\]

To derive probability of false positive \( P_{fp}(t) \), based on the definition in Equation (3), it can be rewritten as

\[
P_{fp}(t) = P\{j \in S_i(t) | H(t)\}
= P\{j \in S_i(t - 1) | H(t)\} \times P\{j \in S_i(t - 1) | j \in S_i(t - 1) \cap H(t)\}
= P_{fp}(t - 1) \times P\{j \text{ is not removed at round } t | j \in S_i(t - 1) \cap H(t)\},
\]

where \( j \) is an honest friend of detector \( i \). To compute the probability that \( j \) is not removed at round \( t \), we first consider the case where round \( t \) is detectable. Considering that a user in an OSN usually has a large number of neighbors and dishonest users only account for a small fraction, we approximate the probability of an honest user in the suspicious set not being removed at round \( t \) as \( \frac{|D(\tau - 1) \cap D(t)|}{|D(\tau - 1)|} \). On the other hand, if round \( t \) is not detectable, then the corresponding probability is simply one. For ease of presentation, we always let \( D(t) = D(t - 1) \) if round \( t \) is not detectable, so the probability can still be expressed as \( \frac{|D(\tau - 1) \cap D(t)|}{|D(\tau - 1)|} \). By substituting it in Equation (8), we have

\[
P_{fp}(t) \approx \prod_{\tau=1}^{t} \frac{|D(\tau - 1) \cap D(\tau)|}{|D(\tau - 1)|},
\]

where \( D(0) \) is initialized as \( N_i \) and \( D(\tau) \) is set as \( D(\tau - 1) \) if \( d(\tau) = 0 \).

Now we focus on the third performance measure \( R \), which denotes the number of rounds needed to shrink the suspicious set until it only contains dishonest users. Note that the suspicious set can shrink at round \( t \) only when this round is detectable, i.e., \( d(t) = 1 \). Therefore,
we first derive the distribution of number of detectable rounds, and denote it by a random variable $D$. Formally, we have

$$
P(D \leq d) = P\{\text{after } d \text{ detectable rounds, all users in the suspicious set are dishonest}\}
= P\{\text{all honest users are removed from the suspicious set after } d \text{ detectable rounds}\}
= (1 - (1 - p_{hc})^d)^{N-k},
$$

where $k$ is the number of dishonest neighbors of detector $i$ and $p_{hc}$ denotes the average probability of an honest user being removed from the suspicious set at each round, i.e., the average probability of an honest user giving correct recommendations at each round.

Based on the distribution of $D$, we can derive the distribution of $R$. Specifically, the conditional distribution $P(R=r|D=d)$ is a negative binomial distribution, so we have

$$
P(R=r) = \sum_{d=r}^{r} \binom{r}{d} (pd)^d (1-p_d)^{r-d} P(D=d),
$$

where $p_d$ denotes the probability of a round being detectable. Based on the distribution of $R$, the expected number of rounds $E[R]$ can be easily derived.

For probabilities $p_{hc}$ and $p_d$, we can estimate them based on the detection history of detector $i$. Specifically, to measure $p_{hc}$ for each honest neighbor $j$ of detector $i$, we first count the number of rounds where user $j$ gives correct recommendations to detector $i$, then use the fraction of rounds where user $j$ gives correct recommendations as an approximation of the corresponding average probability. Finally, we can approximate $p_{hc}$ by taking an average over all honest neighbors of detector $i$. Mathematically, we have

$$
p_{hc} \approx \frac{1}{N-k} \sum_{\text{honest } j \in N_i} \frac{\# \text{ of rounds where } j \text{ gives correct rec.}}{\text{total } \# \text{ of rounds}},
$$

where $k$ denotes the number of dishonest neighbors of detector $i$. With respect to $p_d$, note that $p_d$ equals to the probability that detector $i$ values her purchased product as a trustworthy product in a round and this round is further used for detection. To estimate it, we use a 0-1 random variable $1\{T_i(P_j) = 1\}$ to indicate whether product $P_j$ that is purchased by detector $i$ at round $t$ is a trustworthy product or not, and we have

$$
p_d = p \cdot \lim_{n \to \infty} \frac{\sum_{i=1}^{n} 1\{T_i(P_j) = 1\}}{n}.
$$

**Proof of Theorem 2 in Section 5**

Note that the detection at round $t$ is divided into two steps, the independent detection step and the cooperative detection step. In the independent detection step, detector $i$ shrinks her suspicious set based on her local information, i.e., via Algorithm 1. In the cooperative detection step, detector $i$ further shrinks her suspicious set based on her neighbors’ detection results. We use $C(t)$ to denote the set of neighbors which are removed from the suspicious set in the cooperative detection step at round $t$. Based on Equation (8), probability of false positive can be expressed as follows.

$$
P_{fp}(t) = P_{fp}(t-1) \times P\{j \text{ is not removed at round } t| j \in S(t-1) \cap C(t)\}
= P_{fp}(t-1) P_{IS}(t) P_{CS}(t),
$$

where $j$ an honest friend of detector $i$, $P_{IS}(t)$ and $P_{CS}(t)$ denote the probabilities that an honest user in the suspicious set is not removed in the independent detection step and the cooperative detection step, respectively.

To derive $P_{IS}(t)$, since the suspicious set shrinks based on Algorithm 1 in the independent detection step, we can directly use the result in Theorem 1. We have

$$
P_{IS}(t) \approx \frac{|D(t-1) \cap D(t)|}{|D(t-1)|},
$$

where $D(t)$ is set as $D(t-1)$ if round $t$ is not detectable (i.e., when $d(t) = 0$).

To compute $P_{CS}(t)$ that is the probability that an honest user in the suspicious set is not removed in the cooperative detection step, we first compute the probability of false positive before the cooperative detection step at round $t$, and denote it by $P_{IS}^{fp}(t)$. Mathematically,

$$
P_{IS}^{fp}(t) \approx P_{fp}(t-1) \frac{|D(t-1) \cap D(t)|}{|D(t-1)|}.
$$

Thus, there are $P_{IS}^{fp}(t)(N-k)$ honest users in the suspicious set if the detector has $k$ dishonest neighbors. Since $|C(t)|$ users are removed from the suspicious set in the cooperative detection step, we have

$$
P_{CS}(t) = \frac{P_{IS}^{fp}(t)(N-k) - |C(t)|}{P_{IS}^{fp}(t)(N-k)}.
$$

Now probability of false positive after $t$ rounds can be derived as follows.

$$
P_{fp}(t) \approx P_{fp}(t-1) \frac{|D(t-1) \cap D(t)|}{|D(t-1)|} \frac{(N-k) - |C(t)|}{N-k}.
$$

If $k \ll N$, then probability of false positive $P_{fp}(t)$ after $t$ rounds can be approximated as

$$
P_{fp}(t) \approx P_{fp}(t-1) \frac{|D(t-1) \cap D(t)|}{|D(t-1)|} N - |C(t)|.
$$

Note that if $k \ll N$ does not hold, then probability of false positive in Equation (12) is just an overestimation of Equation (11), so it is still feasible to be used in the termination condition of the complete algorithm.
Proof of Theorem 3 in Section 6

Inspired from the previous analysis, we divide the detection at round \( t \) into three steps: (1) the independent detection step, (2) the cooperative detection step, and (3) the detection step dealing with user churn. Moreover, probability of false positive after the cooperative detection step at round \( t \) can be derived by Equation (11), and we denote it as \( P_{tp}^{CS}(t) \). Since users in \( N(t) \) connect with detector \( i \) and users in \( L(t) \) leave detector \( i \) at round \( t \), suppose that dishonest users only account for a small fraction of the population, there are around \( N(t) - k \) honest users in the neighboring set after round \( t \), where \( N(t-1) \) denotes the number of neighbors of detector \( i \) after round \( t-1 \) and \( N(t) \) is \( N(t-1) + |N(t)| \) honest users in the suspicious set after round \( t \) is \( P_{tp}^{CS}(t) \times (N(t-1)-k) + |N(t)| \times |L(t)| \). Moreover, the number of honest users in the suspicious set after round \( t \) is \( P_{tp}^{CS}(t) \times (N(t-1)-k) + |N(t)| \times |L(t)| \), so probability of false positive after round \( t \) can be computed via \( P_{tp}^{CS}(t) \approx \frac{N(t)-k}{N(t)} \). If \( k \ll N(t) \), then we substitute \( P_{tp}^{CS}(t) \) with the result in Equation (11), we have

\[
P_{tp}(t) = \frac{P_{tp}(t-1) \cdot \left( N(t-1) - |L(t)| \right) \cdot |N(t)|}{N(t)}
\]

Again, if \( k \ll N(t) \) for all \( t \) does not hold, probability of false positive computed via the above equation is just overestimated, and it is still effective to use it to design the termination condition of the complete algorithm.

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