Fuzzy Concepts in Expert Systems

K.S. Leung and W. Lam
Chinese University of Hong Kong

The expert system shell System Z-II handles both exact and inexact reasoning. It allows any combination of fuzzy and normal terms and uncertainties.

Most of today's commercial expert-system building tools use certainty or confidence factors to handle uncertainties in the knowledge or data.[1] But they cannot cope with fuzzy concepts such as *tall*, *good*, or *hot*, which constitute a very significant part of a natural language. In fact, some of these fuzzy concepts have been incorporated into several expert systems, such as Cadiag-2[2] and Fault,[3] which are purposely built from a high-level language for a specific domain of application. In Cadiag-2 the knowledge representation of fuzzy concepts is designed specifically for medical diagnosis. In Fault some fuzzy reasoning is also supported. Several AI programming languages, such as FProlog, also provide mechanisms to handle fuzzy concepts.[4] FProlog is similar to Prolog except that in FProlog a truth value expressed numerically is allowed in a fact. The uncertainty can then be handled automatically by the FProlog interpreter.

This article presents a comprehensive expert-system building tool, called System Z-II, that can deal with exact, fuzzy (or inexact), and combined reasoning, allowing fuzzy and normal terms to be freely mixed in the rules and facts of an expert system. This fully implemented tool has been used to build several expert systems in the fields of student curriculum advisement, medical diagnosis, psychoanalysis, and risk analysis. System Z-II is a rule-based system that employs fuzzy logic and fuzzy numbers for its inexact reasoning. It uses two basic inexact concepts, fuzziness and uncertainty, which are distinct from each other in the system.

Inexact knowledge representation and reasoning

Much human knowledge is vague and imprecise.[5,6] Human thinking and reasoning frequently involve inexact information. Expert systems should therefore be able to cope with such inexact information, from the following possible sources:

- inherent human fuzzy concepts,
- unreliable information,
- matching of similar rather than identical experiences,
- incomplete information, and
- differing (expert) opinions.

Types of inexact knowledge. In System Z-II four types of inexact information have been classified:

(1) Uncertainty occurs when one is not absolutely certain about a piece of information. The degree of certainty is usually represented by a numerical value. Example:

X is a bird. (0.8)
If X is a bird, then it can fly. (0.9)

The certainty factors are 0.8 and 0.9.

(2) Fuzziness occurs when the boundary of a piece of information is not clear-cut:

John is quite young.
If the price is high, then the profit should be good.

Quite young, *high*, and *good* are fuzzy terms.

(3) Uncertainty and fuzziness may occur simultaneously in some situations:

0018-9162/88/0900-0043$01.00 © 1988 IEEE

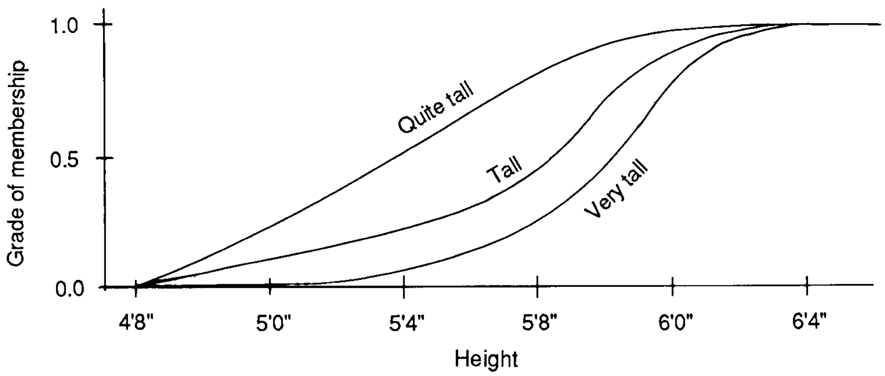


Figure 1. Fuzzy sets of fuzzy terms with modifiers.

John is rather tall. (0.8)
If the price is high, then the profit should be good. (0.9)

The certainty factors are 0.8 and 0.9; *rather tall*, *high*, and *good* are fuzzy terms.

(4) Sometimes the uncertainty can also be fuzzy:

John is very heavy. (around 0.7)

Here, *around 0.7* is the fuzzy uncertainty, and *very heavy* is a fuzzy term.

Dealing with inexact knowledge. The common approaches to inexact knowledge in expert systems[6] are summarized in the following paragraphs.

Bayesian approach. Based on probability theory, the Bayesian approach can deal only with uncertainty. Collecting or estimating all the prior conditional and joint probabilities required for this method is difficult for domain experts. However, it has been suggested that employing conditional independence assumptions can reduce the number of probabilities to be estimated.[6] This approach also depends on the availability of a complete set of hypotheses, and hence its applicability is restricted.[5]

Certainty factors. This approach can deal only with uncertainty. A certainty factor $CF(h, e)$ is a numerical value between zero and one that stands for the degree of confirmation of the hypothesis h based on the evidence e. Certainty factors are used in the Mycin system to handle uncertainty in evidence (facts) and rules.[5] For example:

rule:
  IF X is a bird,
  THEN it can fly. $(CF = 0.9)$
fact:
  X is a bird. $(CF = 0.8)$
conclusion:
  It can fly. $(CF = 0.9 * 0.8 = 0.72)$

One advantage of this approach over probability theory is that it does not require prior probabilities and therefore does not require a large volume of statistical data. Moreover, experts are more comfortable assigning certainty factors to the facts and rules.[5] In fact, certainty factors have been widely adopted in expert system shells such as Emycin and S.1 to handle uncertainty.

Dempster-Shafer theory of evidence. The Dempster-Shafer theory calculates belief functions—measurements of the degree of belief. The theory allows the decomposition of a set of evidence into separate, unrelated sets of evidence; a probability judgment can be separately assigned to each set of evidence.[7] This approach, however, involves many numerical computations, and in the case of a long inference chain the structure of the resulting belief function would be very complex.

Fuzzy logic. Expert systems can use fuzzy logic to handle fuzzy concepts and approximate reasoning.[8] For instance, the fuzzy term *tall* can be defined by the following fuzzy set:

Height	Grade of membership (possibility value)
4 '4 "	0.0
4 '8 "	0.0
5 '0 "	0.1
5 '4 "	0.2
5 '8 "	0.7
6 '0 "	0.9
6 '4 "	1.0
6 '8 "	1.0

These possibility values constitute a possibility distribution of the term *tall*. Modifiers such as *very*, *around*, and *rather* are common in approximate reasoning. We can obtain the possibility distribution of a fuzzy concept like *very tall* or *quite tall* by applying arithmetic operations on the fuzzy set of the basic fuzzy term *tall*. For example, we can calculate the possibility values of each height in the fuzzy set representing the fuzzy concept *very tall* by taking the square of the corresponding possibility values in the fuzzy set of *tall*, as shown in Figure 1.

The rule *If the price is high, then the profit is good*, with fuzzy concepts *high* and *good*, can be modeled by a fuzzy relation R. Let A_1 and A_2 be the fuzzy sets representing the concepts *high* and *good*, respectively. We obtain the fuzzy relation

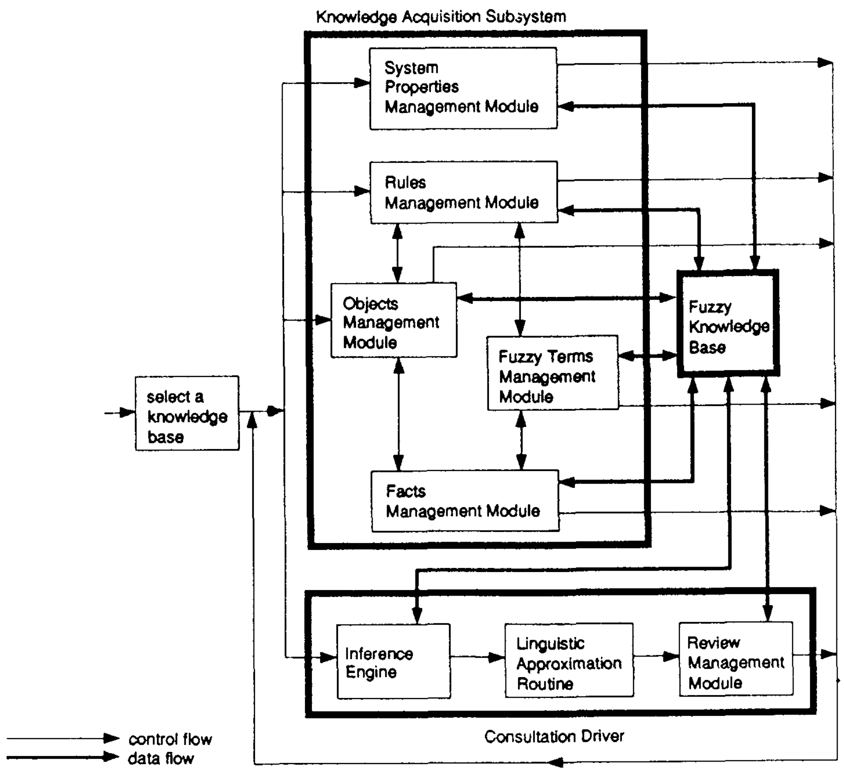


Figure 2. System Z-II's architecture.

R, represented by a matrix, by performing fuzzy operations on A_1 and A_2, expressed as vectors. Many researchers[8] have proposed methods of computing the fuzzy relation R; some of them are presented later in this article in the "Consultation driver" section.

If a fact *The price is very high* matches the rule, the fuzzy concept *very high* can be represented by a fuzzy set F, which we obtain by applying an arithmetic operation (a square operation in this case) on A_1. Then we compute the fuzzy set C, representing the fuzzy term in the conclusion, by applying a fuzzy operator called composition (denoted by o) on F and R: $C = F \circ R$.

The formula for the fuzzy composition is as follows:

composition: $C = F \circ R$

$$\mu_C(x) = \max_{w} (\min(\mu_F(w), \mu_R(w,x)))$$

where $\mu(x)$ is a membership function and w and x are elements in the universe of discourse.[8]

As a result, vector C will indicate *very good* and the conclusion *The profit is very good* will be drawn. This operation forms a simple inference from a fact and a rule both containing fuzzy terms.

System description

System Z-II is an expert system shell that facilitates the construction of rule-based consultation systems. Its major characteristic is that it allows any mix of fuzzy and normal terms as well as uncertainties in the rules and facts. To achieve this task, it employs fuzzy logic to handle inexact reasoning and fuzzy numbers to handle fuzzy uncertainty. Moreover, its menu-driven and restricted natural language interfaces based on fuzzy logic are specifically designed for easy knowledge engineering and consultations.

The development environment of Z-II consists of VAX-Lisp version 2.1 and VAX-Pascal version 3.4 running under VMS version 4.5 on a VAX-11/780 computer. In fact, Z-II is developed mainly in VAX-Lisp with part of its inference engine written in VAX-Pascal to substantially increase execution speed.[9] VAX-Lisp is an implementation of Common Lisp, a highly portable, efficient, and powerful dialect.

Basically, System Z-II consists of three subsystems: the knowledge acquisition subsystem, the consultation driver, and the fuzzy knowledge base (see Figure 2). The components of the knowledge acquisition subsystem include management modules for objects, facts, fuzzy terms, rules, and system properties. These mod-

Table 1. Contents of predefined object slots.

Slot	Contents
TYPE	Indicates type of an object
FUZZY-OR-NOT	Indicates whether values of an object are fuzzy or not
ASK-FIRST-OR-NOT	Indicates whether value of this object is obtained by asking questions first or deducing from known facts and rules
USED-BY-RULES	List of rules whose antecedent parts contain this object
UPDATED-BY-RULES	List of rules whose consequent parts contain this object

ules are responsible for acquiring and managing rules and facts, which may contain any mix of fuzzy and normal terms and uncertainties. The task of the fuzzy knowledge base is to store all these knowledge entities.

The consultation driver consists of three modules: the inference engine, the linguistic approximation routine, and the review management module. The function of the inference engine is to extract the knowledge stored in the fuzzy knowledge base and to make inferences from the respective rules and facts. The linguistic approximation routine maps a set of fuzzy sets onto a set of linguistic expressions or descriptions, translating a fuzzy set into natural language after Z-II has drawn a conclusion. The review management module handles various reviews requested by users.

Knowledge acquisition subsystem

Objects management module. This module creates, modifies, or deletes objects in the system. An object is a basic entity in the system. It is uniquely identified by two elements: an object name and an attribute. For instance, the term *the weight of the body* is represented by the object BODY WEIGHT, with BODY being the object name and WEIGHT being the attribute. An attribute may be empty if the object name is sufficient to describe the object. An object is instantiated to a single value or multiple values during a consultation. It possesses a number of predefined slots that specify its properties. (The use of slots for knowledge representation is similar to the frames approach.) The contents of slots should be given in the knowledge acquisition phase. The objects management module provides routines to manipulate objects as well as their slot contents. The contents of some of the slots are listed in Table 1.

Fuzzy terms management module. Normally, the values of an object are literal strings or numbers. However, if an object is fuzzy (indicated by the FUZZY-OR-NOT slot), its associated values can be fuzzy expressions such as *very tall* and *rather good*. These fuzzy terms are represented by fuzzy sets, and the fuzzy terms management module provides routines to define fuzzy sets for corresponding fuzzy terms. A fuzzy set is effectively a list of numbers. The management module uses Lisp mapping functions to manipulate lists and individual elements in a list, making it easy to implement the primitive fuzzy set operations.

Facts management module. System Z-II has two alternative ways to enter facts. One method is for the user to enter restricted English sentences. The other is invoked by the system asking for information about an object and its certainty step by step. The facts management module can either be invoked by the user or by the inference engine when it finds that some required facts are missing.

A fact is actually a data proposition of this form:

<OBJECT> is <VALUE> (fuzzy/nonfuzzy uncertainty)

The value of a numeric object is a number, while a nonnumeric object contains a string of symbols. If an object is fuzzy, however, its value is a linguistic expression such as *very tall* or *quite good*. *Very tall* and *quite good* are represented by two fuzzy sets obtained by taking the square and square root of the fuzzy sets representing the basic fuzzy concepts *tall* and *good* respectively.

Fuzzy uncertainty is modeled by fuzzy numbers representing the concepts *around 0.8*, *close to 1.0*, and so on. A fuzzy number is actually a real-number fuzzy set that is both convex and normal. The definitions of convex and normal fuzzy sets are given in the sidebar on fuzzy numbers.

Fuzzy numbers, like ordinary numbers, can be used in arithmetic operations (for example, addition and multiplication) that give another fuzzy number as the result. It should be noted that fuzzy uncertainty is optional and the other two options are nonfuzzy uncertainty expressed as ordinary certainty factors and absolute certainty ($CF = 1$). The methods of handling fuzzy uncertainty and fuzzy numbers are explained elsewhere in this article.

Rules management module. A rule is defined as an implication statement expressing the relationship between a set of antecedent propositions and a set of consequent propositions. Attached to each rule is a fuzzy/nonfuzzy uncertainty describing the degree of confidence in the rule. The antecedent part of a rule consists of a single proposition or any combination of two or more propositions connected by either a logical AND or a logical OR. But the consequent part of a rule can contain only a single proposition or multiple propositions with AND conjunctions between them. The reason for this is that the applicability of a rule with OR conjunctions in its consequent part is limited when backward reasoning (chaining) is employed. The following is an example of a rule with multiple propositions:

```
rule01: IF   (the body is well-built
              OR the height is tall)
              AND
              the person is healthy
        THEN  the weight of the person
              is heavy
        WITH CERTAINTY → close
              to 1.0
```

System properties management module. This module is responsible for the manipulation of the system properties of a knowledge base in System Z-II. The following are some system properties:

- GOAL-OBJECTS: specifies goal objects for the knowledge base

- INITIAL-ASK-OBJECTS: specifies objects whose values are to be asked at the start of a consultation

- DOMAIN-DESCRIPTION: stores the descriptions for the domain

Consultation driver

Inference engine. The inference engine of Z-II uses backward chaining to build the appropriate reasoning trees, but uses forward evaluation of the values of the fuzzy terms. It can handle rules with multiple propositions, and it uses evidence combination for cases in which two or more rules have the same consequent proposition.

Reasoning chain. System Z-II adopts backward reasoning during the consulta-

Fuzzy numbers

A fuzzy number is a real-number fuzzy set that is both convex and normal. The following is the definition of a convex fuzzy set F:

$$\forall x, y \in R : \mu_F[\lambda x + (1-\lambda)y] \geq \mu_F(x) \wedge \mu_F(y) \qquad \forall \lambda \in [0,1]$$

where R is the set of real numbers, and x, y, and μ are real numbers. A fuzzy set is normal if and only if the highest value of the degree of membership equals 1.0.

Expert systems can use fuzzy numbers to handle fuzziness or imprecision in real numbers and thus to represent and manipulate linguistic terms such as *near 0.6* and *close to 3*. In System Z-II fuzzy numbers represent the fuzzy uncertainty associated with a rule or a fact.

Fuzzy numbers, like ordinary numbers, can be used in arithmetic operations (addition, multiplication) that give another fuzzy number as the result (see the figure below). The formulas of some fuzzy number arithmetic operators are as follows:

fuzzy number addition +:

$$\mu_{A+B}(z) = \bigvee_{z=x+y} (\mu_A(x) \wedge \mu_B(y))$$

fuzzy number subtraction −:

$$\mu_{A-B}(z) = \bigvee_{z=x-y} (\mu_A(x) \wedge \mu_B(y))$$

fuzzy number multiplication *:

$$\mu_{A*B}(z) = \bigvee_{z=x*y} (\mu_A(x) \wedge \mu_B(y))$$

fuzzy number division /:

$$\mu_{A/B}(z) = \bigvee_{z=x/y} (\mu_A(x) \wedge \mu_B(y))$$

minimum of fuzzy numbers min_fn:

$$\mu_{\mathrm{min\_fn}(A,B)}(z) = \bigvee_{z=x\wedge y} (\mu_A(x) \wedge \mu_B(y))$$

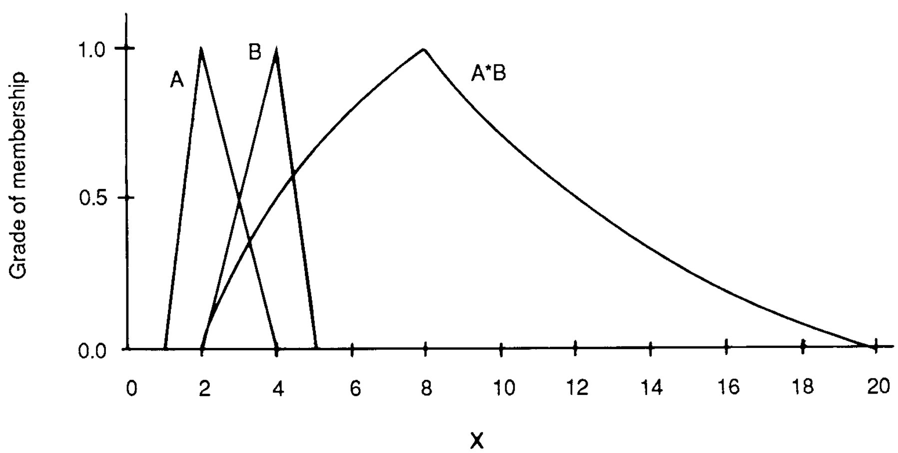
maximum of fuzzy numbers max_fn:

$$\mu_{\mathrm{max\_fn}(A,B)}(z) = \bigvee_{z=x\vee y} (\mu_A(x) \vee \mu_B(y))$$

where

A and B:	fuzzy numbers
μ:	membership distribution function
x, y, and z:	real numbers
V and $\vee$:	taking the maximum
$\wedge$:	taking the minimum

In Z-II, fuzzy numbers are assumed generally to be trapezoidal, and they are implemented as a list of four numbers. It has been found that trapezoidal fuzzy numbers are adequate to capture the fuzzy uncertainties in human intuition. The above fuzzy arithmetic operations are implemented so that they can handle this approximate representation of fuzzy numbers.



Multiplication of two fuzzy numbers.

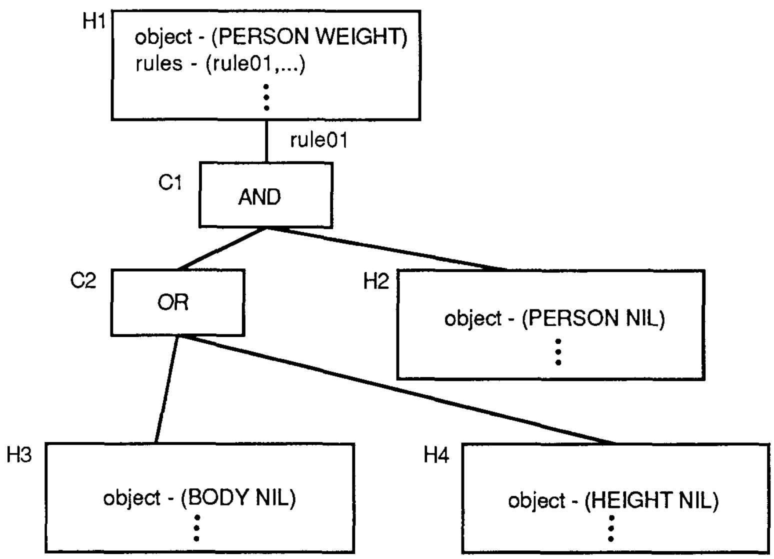


Figure 3. A history tree.

tion process because the questioning is guaranteed to follow the focused goal conclusion. The user may issue queries of the form

What should <OBJECT> be?

e.g., What should the weight of the person be?

The object in the query then becomes the current top-level goal. However, if the user selects an automatic mode of consultation, the system retrieves the objects stored in the GOAL-OBJECTS property of the current knowledge base and then considers each of these objects as the top-level goal. The system begins the reasoning by searching those rules whose consequent propositions have the goal object. This information can be retrieved from the content of the UPDATED-BY-RULES slot of the goal object. Each triggered rule is considered and a history tree is built at the same time.

Suppose there is a rule like the following:

```
rule01: IF   (the body is well-built
              OR the height is tall)
              AND
              the person is healthy
        THEN  the weight of the person
              is heavy
        WITH CERTAINTY → close
              to 1.0
```

If the goal object is *the weight of the person*, the system triggers this rule and starts to examine its antecedent propositions. The resulting history tree is shown in Figure 3.

Based on the history tree, the system examines one of its antecedent objects, BODY NIL, at node H3. If the VALUE-LIST slot of this object is empty, the value of the object is not yet known and it becomes a subgoal object. Therefore, the system tries to obtain the value of the subgoal object by asking a question or by deducing from other rules and facts. The choice depends on the flag stored in the ASK-FIRST-OR-NOT slot of the subgoal object. If the system decides to deduce the value from other rules and facts, the history tree continues to branch downward at node H3. Rules that have existed in ancestor nodes are ignored to prevent infinite looping in building the history tree.

After obtaining the value of the object at the subgoal node H3, the system chooses another subgoal node (H2 or H4) for consideration. The choice depends on two factors. One is the conjunctions between subgoal nodes. The other is whether the value of the object at node H3 is successfully matched by that of the available facts. Finally, the rule is fired if the antecedent part is satisfied completely. Thus, the system can find the value of the goal object by evaluating the rule and the matched fact.

If the objects involved are fuzzy, the inference engine uses fuzzy logic operations, calculating the fuzzy uncertainty of the goal object from the fuzzy uncertainties of the facts and the rules.

Rule evaluation. Suppose there are a rule and a fact:

rule:	IF A is V_1 THEN C is V_2	(FN_1)
fact:	A is $V_1{'}$	(FN_2)
conclusion:	C is $V_2{'}$	(FN_3)

A:	antecedent object
C:	consequent object
FN_1:	fuzzy number denoting uncertainty of the rule
FN_2:	fuzzy number denoting uncertainty of the fact
FN_3:	fuzzy number denoting uncertainty of the conclusion
V_1, V_2, $V_1{'}$, $V_2{'}$:	values

If the object A in the antecedent is nonfuzzy, V_1 and $V_1{'}$ must be the same atomic symbol in order to apply this rule. Therefore, $V_2{'}$ in the conclusion equals V_2, and the fuzzy uncertainty FN_3 of the conclusion is calculated by means of the fuzzy number multiplication of FN_1 and FN_2:

$$FN_3 = FN_1 * FN_2$$

where * denotes a fuzzy number multiplication.[5] The formula for fuzzy number multiplication is given in the sidebar on fuzzy numbers.

If both A and C are fuzzy objects, V_1 and V_2 are represented by fuzzy sets F_1 and F_2 respectively. We can form a fuzzy relation R by performing some fuzzy operations on F_1 and F_2. The default method adopted in System Z-II for forming the fuzzy relation R is the R_{SG} approach proposed by Mizumoto, Fukami, and Tanaka,[8] which has been found to be closer to human intuition and reasoning than other methods. However, the fuzzy relation can be selected from other options available in Z-II, such as R_S and R_G, or it can even be supplied by the user. The following are the three approaches to the fuzzy relation available in Z-II:

$$R_S = F_1 \times V \rightarrow_S U \times F_2$$
$$R_G = F_1 \times V \rightarrow_G U \times F_2$$
$$R_{SG} = (F_1 \times V \rightarrow_S U \times F_2) \wedge (\sim F_1 \times V \rightarrow_G U \times \sim F_2)$$

where

$$\mu_{F_1}(u) \rightarrow_S \mu_{F_2}(v) = \begin{cases} 1 & \text{if } \mu_{F_1}(u) \le \mu_{F_2}(v) \\ 0 & \text{if } \mu_{F_1}(u) > \mu_{F_2}(v) \end{cases}$$

$$\mu_{F_1}(u) \rightarrow_G \mu_{F_2}(v) = \begin{cases} 1 & \text{if } \mu_{F_1}(u) \le \mu_{F_2}(v) \\ \mu_{F_2}(v) & \text{if } \mu_{F_1}(u) > \mu_{F_2}(v) \end{cases}$$

U and V: the universe of discourse of F_1 and F_2 respectively
u and v: the elements of a fuzzy set
$\wedge$: intersection of two fuzzy relations
$\sim$: complement of a fuzzy set
μ : membership function
$\times$: Cartesian product of two fuzzy sets

V_1' in the fact should also be a fuzzy value represented by a fuzzy set F_1'. The fuzzy set F_2' of V_2' in the conclusion is obtained by applying a fuzzy composition operation (denoted by o) on F_1' and R: $F_2' = F_1' \circ R$. The calculation of the fuzzy uncertainty FN_3 of the conclusion is the same.

If A is fuzzy and C is nonfuzzy, V_2' in the conclusion must equal V_2. However, the fuzzy uncertainty FN_3 is obtained by fuzzy multiplication of FN_1, FN_2, and the similarity M between F_1 and F_1', which are the fuzzy sets of V_1 and V_1' respectively:

$$FN_3 = (FN_1 * FN_2 * M)$$

The similarity M is calculated by the following algorithm:

IF $N(F_1|F_1') > 0.5$
THEN $M = P(F_1|F_1')$
ELSE $M = (N(F_1|F_1') + 0.5) * P(F_1|F_1')$

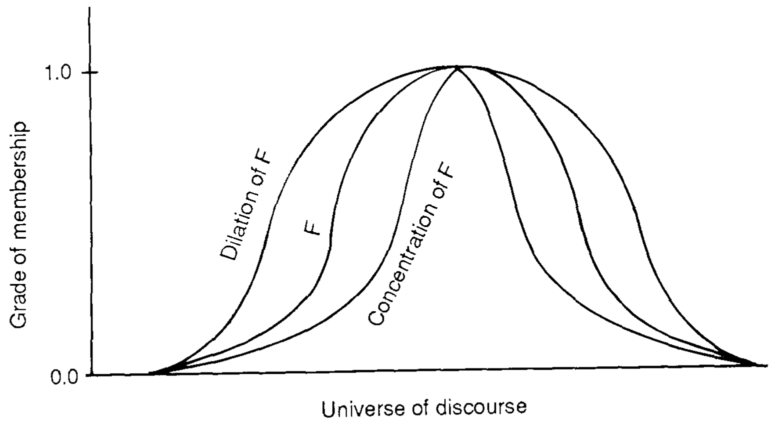


Figure 4. Dilation and concentration of F.

where $P(F_1|F_1')$ is the possibility of the fuzzy data F_1' given the fuzzy pattern F_1, and $N(F_1|F_1')$ is the necessity of the fuzzy data F_1' given the fuzzy pattern F_1.
The following are the formulas of the possibility and necessity measures between the fuzzy data and the fuzzy pattern:

possibility:
$$P(F_1|F_1') = \max(\min(\mu_{F_1}(w), \mu_{F_1'}(w)))$$

necessity:
$$N(F_1|F_1') = 1 - P(\tilde{F}_1|F_1')$$

where μ is the membership distribution function, w is the element in the universe of discourse of the above fuzzy sets, and $\tilde{F}_1$ is the complement of F_1.

The possibility between the fuzzy pattern F_1 and the fuzzy data F_1' gives the maximum of their intersection and measures to what extent they overlap. Under normal circumstances the necessity reflects the following relationships between two fuzzy sets:

$N(F_1|F_1') > 0.5 <=>$ F_1' is a concentration of F_1
$N(F_1|F_1') = 0.5 <=>$ F_1' is a duplicate of F_1
$N(F_1|F_1') < 0.5 <=>$ F_1' is a dilation of F_1

As shown in Figure 4, if F_1' is a concentration of F_1, it means that F_1' has a more concentrated or narrower distribution than F_1. If V_1 and V_1' are the fuzzy terms in the rule and the matched fact respectively, then F_1 and F_1' represent two similar fuzzy concepts such as *good* and *very good* respectively. However, the more concentrated distribution of F_1' represents a more strongly expressed or stressed term than that of F_1. For a dilation the situation is exactly the opposite.

On the other hand, the similarity measures how similar two fuzzy concepts represented by the two fuzzy sets are. When $N(F_1|F_1')$ is larger than 0.5, the similarity becomes saturated and is forced to equal the possibility (usually equal to one for two similar concepts). Examples:

(1) The fuzzy data is a concentration of the fuzzy pattern (i.e., necessity > 0.5).

rule: IF X is tall, THEN X should be chosen as a member of the basketball team. $(CF_1 = 1.0)$
fact: X is very tall. $(CF_2 = 1.0)$
conclusion: X is chosen as a member of the basketball team. $(CF_3 = y)$

A sample case

The sample expert system described here illustrates the main features of System Z-II. Designed to help students choose their university department, this expert system has about 60 rules in its knowledge base. The following are some typical rules used in the system (the complete list appears later in this sidebar, under "Rules of the sample expert system"):

RULE CODE: phy-r5
IF your interest in reading science books or magazines is high
THEN your interest in investigating science should be high
WITH CERTAINTY → 0.9

RULE CODE: med-r1
IF your overall examination performance in medicine is good
THEN my recommendation for your choice of a university department should be medicine
WITH CERTAINTY → 0.9

RULE CODE: test-r1
IF the mark you obtained in the English Usage Test is <50
THEN your English result should be bad
WITH CERTAINTY → 1.0

RULE CODE: exam-r3
IF your physics result is very good AND
your mathematics result is good
THEN your overall examination performance in physics should be good
WITH CERTAINTY → 1.0

RULE CODE: soc-r4
IF your interest in analyzing human behavior is high
THEN your overall interest in social science should be high
WITH CERTAINTY → roughly 0.85

Rule phy-r5 has the fuzzy concept *high* in its antecedent and consequence. Rule med-r1 has the fuzzy concept *good* in its antecedent and a nonfuzzy consequence. Rule test-r1 has a fuzzy consequence and a nonfuzzy antecedent, which is also a numeric-comparison logic control. Rule exam-r3 demonstrates multiple propositions in its antecedent. Some of the rules are not absolutely certain. Thus, a certainty factor, which can be fuzzy or nonfuzzy, is attached to each rule.

During a consultation, the system requests the user to enter the appropriate fuzzy or nonfuzzy facts. After obtaining enough facts, the system presents its conclusions. For a multivalued goal, the conclusions are given in descending order according to their certainties. The following is a sample consultation, including facts and conclusions:

** The following are the facts you entered:

1. The fact that you have a risk-taking personality is false. (1.0)
2. Your interest in the business environment is very low. (about 0.9)
3. Your Chinese result is fair. (1.0)
4. The mark you obtained in the English Usage Test is 76.0. (1.0)
5. Your mathematics result is good. (1.0)
6. The preferred career as a teacher is true. (1.0)
7. The preferred career as a researcher is true. (1.0)
8. Your interest in analyzing human behavior is medium. (around 0.8)
9. Your interest in reading fiction is more or less low. (1.0)
10. The preferred career as a doctor is false. (1.0)
11. Your interest in analyzing the human body is low. (0.7)
12. Your chemistry result is fair. (1.0)
13. Your biology result is fair. (1.0)
14. Your physics result is very good. (1.0)
15. Your interest in manipulating mathematical symbols is rather high. (1.0)
16. The preferred career as an electronics engineer is false. (1.0)
17. Your interest in building electronic kits is medium. (1.0)
18. The preferred career as a programmer is true. (1.0)
19. Your interest in writing computer programs is high. (1.0)
20. Your interest in computers is high. (1.0)
21. Your interest in reading science books or magazines is very high. (1.0)
22. Your interest in performing chemical experiments is medium. (1.0)
23. Your interest in analyzing chemical substances is more or less high. (0.9)
24. Your interest in observing animals is high. (1.0)
25. Your interest in observing plants is high. (1.0)

** After analyzing your responses, Z-II makes the following conclusions in preference order for your choice of a university department:

It is 0.97 certain that — the department of physics
It is 0.95 certain that — the department of computer science
It is 0.93 certain that — the department of mathematics
It is 0.90 certain that — the department of electronics
It is 0.86 certain that — the department of biology
It is 0.72 certain that — the department of chemistry
It is very close to 0.70 certain that — the faculty of arts
It is very close to 0.69 certain that — the faculty of social science
It is close to 0.49 certain that — the faculty of business
It is 0.48 certain that — the faculty of medicine

** It is 0.9 certain that the chance of your entering the physics department is fuzzily very high.

In this consultation, two goals can be changed easily in the expert system. The first one is *department selection*, which is a multivalued nonfuzzy goal; the other is *the chance of entering the physics department*, a single-valued fuzzy goal. The first goal mainly depends on factors such as the student's examination performance, the student's suitability for a future job in the field, and the student's overall interests. However, the second goal depends only on the performance in the physics examination, and its conclusion is a fuzzy term *fuzzily very high*. The membership distribution of the fuzzy set of this term is similar to that of *very high* except that it has a much gentler slope.

Rules of the sample expert system

RULE CODE: exam-r1
IF your biology result is very good AND
your chemistry result is good AND
(your mathematics result is fair or good OR
your physics result is fair or good)
THEN your overall examination performance in biology should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r2
IF your chemistry result is very good AND (your mathematics result is fair or good OR your physics result is fair or good)
THEN your overall examination performance in chemistry should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r3
IF your physics result is very good AND your mathematics result is good
THEN your overall examination performance in physics should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r4
IF your mathematics result is very good AND your physics result is good
THEN your overall examination performance in computer science should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r5
IF your mathematics result is good AND your physics result is good
THEN your overall examination performance in electronics should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r6
IF your mathematics result is very good AND your physics result is fair or good
THEN your overall examination performance in mathematics should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r7
IF your chemistry result is very good AND your biology result is very good AND your mathematics result is good AND your physics result is good
THEN your overall examination performance in medicine should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r8
IF your Chinese result is good AND your English result is good
THEN your overall examination performance in arts should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r9
IF your Chinese result is good AND your English result is good AND your mathematics result is fair or good
THEN your overall examination performance in social science should be good
WITH CERTAINTY → 1.0

RULE CODE: exam-r10
IF your Chinese result is good AND your English result is good AND your mathematics result is good
THEN your overall examination performance in business should be good
WITH CERTAINTY → 1.0

RULE CODE: phy-r1
IF your overall examination performance in physics is good
THEN your chance of entering physics should be high AND my recommendation for your choice of a university department should be physics
WITH CERTAINTY → 0.9

RULE CODE: phy-r2
IF your overall interest in physics is high
THEN my recommendation for your choice of a university department should be physics
WITH CERTAINTY → 0.6

RULE CODE: phy-r3
IF the overall suitability of a future job in physics is good
THEN my recommendation for your choice of a university department should be physics
WITH CERTAINTY → 0.4

RULE CODE: phy-r4
IF your interest in investigating science is high
THEN your overall interest in physics should be high
WITH CERTAINTY → 1.0

RULE CODE: phy-r5
IF your interest in reading science books or magazines is high
THEN your interest in investigating science should be high
WITH CERTAINTY → 0.9

RULE CODE: mth-r1
IF your overall examination performance in mathematics is good
THEN my recommendation for your choice of a university department should be mathematics
WITH CERTAINTY → 0.9

RULE CODE: mth-r2
IF your overall interest in mathematics is high
THEN my recommendation for your choice of a university department should be mathematics
WITH CERTAINTY → 0.6

RULE CODE: mth-r3
IF the overall suitability of a future job in mathematics is good
THEN my recommendation for your choice of a university department should be mathematics
WITH CERTAINTY → 0.4

RULE CODE: mth-r4
IF your interest in manipulating mathematical symbols is high
THEN your overall interest in mathematics should be high
WITH CERTAINTY → 0.85

RULE CODE: csc-r1
IF your overall examination performance in computer science is good
THEN my recommendation for your choice of a university department should be computer science
WITH CERTAINTY → 0.9

RULE CODE: csc-r2
IF your overall interest in computer science is high
THEN my recommendation for your choice of a university department should be computer science
WITH CERTAINTY → 0.6

RULE CODE: csc-r3
IF the overall suitability of a future job in computer science is good
THEN my recommendation for your choice of a university department should be computer science
WITH CERTAINTY → 0.4

RULE CODE: csc-r4
IF your interest in computers is high
THEN your overall interest in computer science should be high
WITH CERTAINTY → 0.8

RULE CODE: csc-r5
IF your interest in writing computer programs is high
THEN your overall interest in computer science should be high
WITH CERTAINTY → 0.8

RULE CODE: bio-r1
IF your overall examination performance in biology is good
THEN my recommendation for your choice of a university department should be biology
WITH CERTAINTY → 0.9

RULE CODE: bio-r2
IF your overall interest in biology is high
THEN my recommendation for your choice of a university department should be biology
WITH CERTAINTY → 0.6

RULE CODE: bio-r3
IF the overall suitability of a future job in biology is good
THEN my recommendation for your choice of a university department should be biology
WITH CERTAINTY → 0.4

RULE CODE: bio-r4
IF your interest in investigating living things is high
THEN your overall interest in biology should be high
WITH CERTAINTY → 1.0

RULE CODE: bio-r5
IF your interest in observing plants is high
THEN your interest in investigating living things should be high
WITH CERTAINTY → 0.8

RULE CODE: bio-r6
IF your interest in observing animals is high
THEN your interest in investigating living things should be high
WITH CERTAINTY → 0.8

RULE CODE: chm-r1
IF your overall examination performance in chemistry is good
THEN my recommendation for your choice of a university department should be chemistry
WITH CERTAINTY → 0.9

RULE CODE: chm-r2
IF your overall interest in chemistry is high
THEN my recommendation for your choice of a university department should be chemistry
WITH CERTAINTY → 0.6

RULE CODE: chm-r3
IF the overall suitability of a future job in chemistry is good
THEN my recommendation for your choice of a university department should be chemistry
WITH CERTAINTY → 0.4

RULE CODE: chm-r4
IF your interest in analyzing chemical substances is high
THEN your overall interest in chemistry should be high
WITH CERTAINTY → 0.8

RULE CODE: chm-r5
IF your interest in performing chemical experiments is high
THEN your overall interest in chemistry should be high
WITH CERTAINTY → 0.8

RULE CODE: ele-r1
IF your overall examination performance in electronics is good
THEN my recommendation for your choice of a university department should be electronics
WITH CERTAINTY → 0.9

RULE CODE: ele-r2
IF your overall interest in electronics is high
THEN my recommendation for your choice of a university department should be electronics
WITH CERTAINTY → 0.6

RULE CODE: ele-r3
IF the overall suitability of a future job in electronics is good
THEN my recommendation for your choice of a university department should be electronics
WITH CERTAINTY → 0.4

RULE CODE: ele-r4
IF your interest in building eletronic kits is high
THEN your overall interest in electronics should be high
WITH CERTAINTY → 0.9

RULE CODE: art-r1
IF your overall examination performance in arts is good
THEN my recommendation for your choice of a university department should be arts
WITH CERTAINTY → 0.9

RULE CODE: art-r2
IF your overall interest in arts is high
THEN my recommendation for your choice of a university department should be arts
WITH CERTAINTY → 0.6

RULE CODE: art-r3
IF the overall suitability of a future job in arts is good
THEN my recommendation for your choice of a university department should be arts
WITH CERTAINTY → 0.4

RULE CODE: art-r4
IF your interest in reading fiction is high
THEN your overall interest in arts should be high
WITH CERTAINTY → around 0.9

RULE CODE: ba-r1
IF your overall examination performance in business is good
THEN my recommendation for your choice of a university department should be business
WITH CERTAINTY → 0.9

RULE CODE: ba-r2
IF your overall interest in business is high
THEN my recommendation for your choice of a university department should be business

WITH CERTAINTY → 0.6

RULE CODE: ba-r3
IF the overall suitability of a future job in business is good
THEN my recommendation for your choice of a university department should be business
WITH CERTAINTY → 0.4

RULE CODE: ba-r4
IF your interest in the business environment is high
THEN your overall interest in business should be high
WITH CERTAINTY → 0.9

RULE CODE: med-r1
IF your overall examination performance in medicine is good
THEN my recommendation for your choice of a university department should be medicine
WITH CERTAINTY → 0.9

RULE CODE: med-r2
IF your overall interest in medicine is high
THEN my recommendation for your choice of a university department should be medicine
WITH CERTAINTY → 0.6

RULE CODE: med-r3
IF the overall suitability of a future job in medicine is good
THEN my recommendation for your choice of a university department should be medicine
WITH CERTAINTY → 0.4

RULE CODE: med-r4
IF your interest in analyzing the human body is high
THEN your overall interest in medicine should be high
WITH CERTAINTY → 0.95

RULE CODE: soc-r1
IF your overall examination performance in social science is good
THEN my recommendation for your choice of a university department should be social science
WITH CERTAINTY → 0.9

RULE CODE: soc-r2
IF your overall interest in social science is high
THEN my recommendation for your choice of a university department should be social science
WITH CERTAINTY → 0.6

RULE CODE: soc-r3
IF the overall suitability of a future job in social science is good
THEN my recommendation for your choice of a university department should be social science
WITH CERTAINTY → 0.4

RULE CODE: soc-r4
IF your interest in analyzing human behavior is high
THEN your overall interest in social science should be high
WITH CERTAINTY → roughly 0.85

RULE CODE: job-r1
IF the preferred career as a teacher is desirable OR
the preferred career as a researcher is desirable
THEN the overall suitability of a future job in biology should be good AND
the overall suitability of a future job in chemistry should be good AND
the overall suitability of a future job in physics should be good AND
the overall suitability of a future job in mathematics should be good AND
the overall suitability of a future job in arts should be good AND
the overall suitability of a future job in social science should be good
WITH CERTAINTY → 1.0

RULE CODE: job-r2
IF the preferred career as a programmer is desirable
THEN the overall suitability of a future job in computer science should be good
WITH CERTAINTY → 1.0

RULE CODE: job-r3
IF the preferred career as an electronics engineer is desirable
THEN the overall suitability of a future job in electronics should be good
WITH CERTAINTY → 1.0

RULE CODE: job-r4
IF the preferred career as a doctor is desirable
THEN the overall suitability of a future job in medicine should be good
WITH CERTAINTY → 1.0

RULE CODE: job-r5
IF the fact that you have a risk-taking personality is true
THEN the overall suitability of a future job in business should be good
WITH CERTAINTY → 0.95

RULE CODE: test-r1
IF the mark you obtained in the English Usage Test is <50.0
THEN your English result should be bad
WITH CERTAINTY → 1.0

RULE CODE: test-r2
IF the mark you obtained in the English Usage Test is <70.0 AND
the mark you obtained in the English Usage Test is ≥ 50.0
THEN your English result should be fair
WITH CERTAINTY → 1.0

RULE CODE: test-r3
IF the mark you obtained in the English Usage Test is <80.0 AND
the mark you obtained in the English Usage Test is ≥ 70.0
THEN your English result should be quite good
WITH CERTAINTY → 1.0

RULE CODE: test-r4
IF the mark you obtained in the English Usage Test is <90.0 AND
the mark you obtained in the English Usage Test is ≥ 80.0
THEN your English result should be good
WITH CERTAINTY → 1.0

RULE CODE: test-r5
IF the mark you obtained in the English Usage Test is ≥ 90.0
THEN your English result should be very good
WITH CERTAINTY → 1.0

Therefore, F_1 is the fuzzy set of *tall*, while F_1' is the fuzzy set of *very tall*. If the membership distributions of F_1 and F_1' equal that in Figure 1, then

$$N(F_1|F_1') = 0.6 \qquad (> 0.5)$$
$$P(F_1|F_1') = 1.0$$

Based on the above algorithm,

$$M = P(F_1|F_1') = 1.0$$

As a result, the certainty factor of the conclusion,

$$y = CF_1 * CF_2 * M = 1.0 * 1.0 * 1.0 = 1.0$$

(2) If the fuzzy data is a dilation of the fuzzy pattern (i.e., necessity < 0.5), then the similarity should depend on the necessity and possibility measures.

Assume the fact in the above example is changed to

fact: X is quite tall. ($CF_2 = 1.0$)

Now, F_1' is the fuzzy set representing *quite tall* and if its membership distribution equals that in Figure 1, then

$$N(F_1|F_1') = 0.3 \qquad (< 0.5)$$
$$P(F_1|F_1') = 1.0$$

Based on the above algorithm,

$$M = (N(F_1|F_1') + 0.5) * P(F_1|F_1') = (0.3 + 0.5) * 1.0 = 0.8$$

As a result, the certainty factor of the conclusion,

$$y = 1.0 * 1.0 * 0.8 = 0.8$$

Rules with multiple propositions. In Z-II the consequent part of a rule can contain only multiple propositions ($C_1, C_2, \ldots, C_n$) with AND conjunctions between them. They can be treated as multiple rules with a single conclusion. So the following rule:

IF antecedent-propositions,
THEN C_1 AND C_2 AND . . . C_n

is equivalent to the following rules:

IF antecedent-propositions, THEN C_1
IF antecedent-propositions, THEN C_2
•
•
•
IF antecedent-propositions, THEN C_n

Therefore, only the problem of multiple propositions in the antecedent with a single proposition in the consequence needs to be considered. If the object in the consequent proposition is nonfuzzy, no special treatment is needed. However, if the consequent proposition is fuzzy, the fuzzy set of the value V_3' in the conclusion is calculated with the following two basic algorithms[10]:

(1) rule: IF A_1 AND A_2, THEN C is V_3
facts: A_1', A_2'
conclusion: C is V_3'
algorithm: The fuzzy set representing V_3' in the conclusion C is obtained by taking fuzzy union of the fuzzy sets of F_1 and F_2
where the fuzzy set F_1 is obtained from the composition operation on the single rule (IF A_1, THEN C is V_3) and the fact A_1', while the fuzzy set F_2 is obtained from the composition operation on the single rule (IF A_2, THEN C is V_3) and the fact A_2'. Union operation is used on F_1 and F_2 because they have an OR relation between them after we break up the rule [$(A_1$ AND $A_2) \rightarrow C$] into [$(A_1 \rightarrow C)$ OR $(A_2 \rightarrow C)$] by the distribution law in classic logic.

(2) rule: IF A_1 OR A_2, THEN C is V_3
facts: A_1', A_2'
conclusion: C is V_3'
algorithm: The same as above except that fuzzy intersection rather than union should be applied on the fuzzy sets of F_1 and F_2

where

A_1, A_2: antecedent propositions that can be single or multiple
C: object in the consequent proposition
A_1', A_2': data propositions (facts)
V_3, V_3': values
fuzzy intersection: IF D is the fuzzy intersection of F_1 and F_2, THEN $\mu_D(x) = \min(\mu_{F_1}(x), \mu_{F_2}(x))$
fuzzy union: IF D is the fuzzy union of F_1 and F_2, THEN $\mu_D(x) = \max(\mu_{F_1}(x), \mu_{F_2}(x))$
D, F_1, F_2: fuzzy sets

The above two algorithms can be applied repeatedly to handle any combination of antecedent propositions. For instance:

rule: IF (the raw material cost is low OR the production cost is low) AND sales are high
THEN the profit should be good
facts: The raw material cost is very low
The production cost is low
Sales are rather high

where *low*, *high*, *good*, *very low*, and *rather high* are fuzzy concepts.

Let F_1 be the fuzzy set obtained by making an inference from the single rule
IF the raw material cost is low THEN the profit should be good
and the fact
The raw material cost is very low ;

F_2 be the fuzzy set obtained by making an inference from the single rule
IF the production cost is low THEN the profit should be good
and the fact
The production cost is low ; and

F_3 be the fuzzy set obtained by making an inference from the single rule
IF sales are high THEN the profit should be good
and the fact
Sales are rather high

The fuzzy set F representing the fuzzy value of the object *profit* in the conclusion is determined as follows:

F = fuzzy union between F_{12} and F_3, where
F_{12} = fuzzy intersection between F_1 and F_2

As a result, F will indicate the fuzzy concept *good* and the conclusion *The profit is good* is drawn.

The fuzzy uncertainty of the conclusion deduced from rules with multiple-antecedent propositions is calculated by means of fuzzy number arithmetic operators[1] in formulas used by Mycin's CF model. For example:

rule:	IF A_1 and A_2 then C	(FN_R)
facts:	A_1'	(FN_1)
	A_2'	(FN_2)
conclusion: C'		(FN_C)

FN_C = (min_fn (FN_1, FN_2)) * FN_R

A_1,A_2:	antecedent propositions that can be single or multiple
C:	consequent proposition
C':	conclusion
FN_R:	fuzzy uncertainty of the rule
FN_1,FN_2:	fuzzy uncertainties of the facts
FN_C:	fuzzy uncertainty of the conclusion
min_fn:	take the minimum of two fuzzy numbers
*:	fuzzy number multiplication
A_1',A_2':	data propositions (facts)

If logical OR is used, the calculation is the same except that the fuzzy maximum is taken rather than the minimum. For any combination of antecedent propositions, the two calculations can be applied repeatedly to handle fuzzy uncertainties corresponding to the matched facts and the rule. (For fuzzy operations, see the sidebar on fuzzy numbers.)

Evidence combination. In some cases, two or more rules have the same consequent proposition. Each of these rules with matched facts can be treated as contributing evidence toward the conclusion. A conclusion C_R can be drawn from the evidence contributed by these rules and facts. For instance:

rules:	r_1—if A_1 then C	
	r_2—if A_2 then C	
facts:	—A_1',A_2'	
conclusion:	C_R, obtained from	
	—C'	(FN_1)
and	—C''	(FN_2)

where

r_1,r_2:	rule codes
A_1,A_2:	antecedent propositions
C:	consequent proposition
C',C'':	conclusions from r_1 & A_1' and r_2 & A_2' respectively
FN_1,FN_2:	fuzzy uncertainties of the conclusion

If the object involved in the consequent proposition is fuzzy, the fuzzy sets corresponding to conclusions C' and C'', obtained from performing approximate reasoning on the respective rules and facts, are combined by taking the fuzzy intersection between them to obtain the fuzzy set corresponding to the combined conclusion C_R. The operation can be applied repeatedly if there are more than two rules with respective matching facts but the same consequent proposition.

The fuzzy uncertainties of the respective conclusions C' and C'' are also aggregated to form an overall uncertainty FN_R for C_R. Basically, two uncertainties are considered at each time and combined according to the following formula, similar to the evidence combination formula in Mycin's CF model:

$$FN_R = FN_1 + FN_2 - (FN_1 * FN_2)$$

where

FN_R:	the combined fuzzy uncertainty
FN_1,FN_2:	fuzzy uncertainties of C' and C'' respectively
+ :	fuzzy number addition
− :	fuzzy number subtraction
* :	fuzzy number multiplication

If there are more than two rules with the same consequent proposition, this formula is repeatedly applied until an overall fuzzy certainty is obtained.

Linguistic approximation routine. Linguistic approximation is a process that maps the set of fuzzy sets onto a set of linguistic values or expressions. In Z-II this process is needed for two purposes. One is to find the corresponding verbal descriptions of fuzzy sets representing fuzzy values. The other is to get the linguistic descriptions of fuzzy numbers representing fuzzy uncertainties, which is an original idea presented in this article.

The technique adopted in the linguistic approximation makes use of two factors: the imprecision and the location of a fuzzy set. The *imprecision* of a fuzzy set is defined as the sum of membership values; the *location* is the center of gravity. The possibility distribution of each linguistic value can be uniquely identified by the imprecision and location of a fuzzy set; the corresponding linguistic value can be matched and selected accordingly.

Review management module. A user can review the case data (facts) at any time during the consultation process and can modify case data after a review. The review management module provides routines to monitor and trace the relevant rules and facts.

This module is also responsible for tracing the reasoning chain when explanations are required. The system provides two types of explanations: *why* a fact is required by the system and *how* a fact is established.

Finally, this module can handle *what-if* reviews, which find out what conclusions will be deduced if certain facts are changed.

Fuzzy knowledge base

The fuzzy knowledge base is responsible for storing the knowledge entities, such as the objects, rules, and fuzzy terms of a knowledge base, acquired through the knowledge acquisition subsystem. These knowledge entities, representing expertise, provide information that enables the inference engine to perform consultations.

The fuzzy knowledge base is implemented with hash tables and property lists. These two data structures provided by Common Lisp are suitable for implementing expert systems in general. In System Z-II several hash tables are used to store objects, fuzzy terms, and rules. Accessing data from a hash table is efficient because the built-in hashing function is transparent to users.[9] Property lists store the slots containing various types of knowledge entities.

The most distinctive feature of the fuzzy knowledge base is that it also stores fuzzy sets representing fuzzy terms. Each fuzzy set is implemented as a list of numbers that represent grades of membership (possibility values) on an imaginary psychological continuum with an interval scale.

To facilitate better knowledge engineering and simulation of human reasoning, we have built fuzzy concepts and inexact reasoning into System Z-II, an expert system shell based on fuzzy logic and fuzzy numbers. The added features of fuzzy concepts and inexact reasoning are particularly indispensable for building expert systems in application areas such as risk analysis and psychoanalysis because imprecision and fuzziness are always present in the natural-language expressions of domain experts and end users in these fields. The system also exploits the power of fuzzy logic in the natural-language user interface. Z-II can handle both fuzziness and uncertainty, the two basic inexact concepts. Fuzzy sets and relations deal with the fuzziness in approximate reasoning, while fuzzy numbers manipulate the uncertainty. The system gains power by allowing any mix of fuzzy and normal terms, numeric-comparison logic controls, and uncertainties.

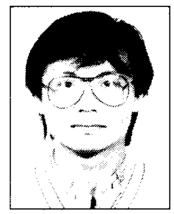
System Z-II has been used to construct several expert systems in university department selection, medical diagnosis, psychoanalysis, and risk analysis. These systems were built with the aid of many experts, who found it natural and convenient to express their knowledge by means of the fuzzy concepts supported by this tool. The satisfactory performance of these systems has demonstrated the feasibility and effectiveness of introducing fuzzy concepts into expert systems.□

References

1. A. Kaufmann and M.M. Gupta, *Introduction to Fuzzy Arithmetic: Theory and Applications*, Van Nostrand Reinhold, New York, 1985.
2. K.P. Adlassnig and G. Kolarz, "Representation and Semiautomatic Acquisition of Medical Knowledge in Cadiag-1 and Cadiag-2," *Computers and Biomedical Research*, Vol. 19, 1986, pp. 63-79.
3. T. Whalen, B. Schott, and F. Ganoe, "Fault Diagnosis in a Fuzzy Network," *Proc. 1982 Int'l Conf. Cybernetics and Society*, IEEE Press, New York, 1982.
4. T.P. Martin, J.F. Baldwin, and B.W. Pilsworth, "The Implementation of FProlog—A Fuzzy Prolog Interpreter," *Fuzzy Sets and Systems*, Vol. 23, 1987, pp. 119-129.
5. B.G. Buchanan and E.H. Shortliffe, *Rule-Based Expert Systems*, Addison-Wesley, Reading, Mass., 1984.
6. J.L. O'Neill, "Plausible Reasoning," *Australian Computer J.*, Vol. 19, No. 1, 1987, pp. 2-15.
7. G. Shafer, *A Mathematical Theory of Evidence*, Princeton University Press, Princeton, N.J., 1976.
8. M. Mizumoto, S. Fukami, and K. Tanaka, "Some Methods of Fuzzy Reasoning," in *Advances in Fuzzy Set Theory and Applications*, M.M. Gupta, R.K. Ragade, and R.R. Yager, eds., North-Holland, Amsterdam, 1979, pp. 117-136.
9. K.S. Leung and W. Lam, "The Implementation of Fuzzy Knowledge-Based System Shells," *Proc. Tencon 87, IEEE Region 10 Conf.*, pp. 650-654.
10. T. Whalen and B. Schott, "Decision Support with Fuzzy Production Systems," in *Advances in Fuzzy Sets, Possibility Theory and Applications*, P.P. Wang, ed., Plenum Press, 1982.

K.S. Leung is a lecturer in the department of computer science at the Chinese University of Hong Kong. His research interests lie in the areas of expert systems and fuzzy logic applications in artificial intelligence.

Leung received his BS and PhD degrees from the University of London in 1977 and 1980, respectively. He is a member of IEEE, BCS, and IEE.



Raymond W. Lam is a research student in the department of computer science at the Chinese University of Hong Kong. His current research interests include expert systems and artificial intelligence.

Lam graduated from the Chinese University of Hong Kong with a BS degree in computer science.